Discrete Optimization 22 (2016) 6–19

Contents lists available at ScienceDirect

Discrete Optimization

www.elsevier.com/locate/disopt

On complexities of minus domination

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ARTICLE INFO

Article history:
Available online 11 May 2016

Keywords:
Domination
Minus domination
Fixed-parameter tractable
d-degenerate graph
Chordal graph

ABSTRACT

A function $f : V \to \{-1, 0, 1\}$ is a minus-domination function of a graph $G = (V, E)$ if the values over the vertices in each closed neighborhood sum to a positive number. The weight of f is the sum of $f(x)$ over all vertices $x \in V$. In the minus-domination problem, one tries to minimize the weight of a minus-domination function. In this paper, we show that (1) the minus-domination problem is fixed-parameter tractable for d-degenerate graphs when parameterized by the size of the minus-dominating set and by d, where the size of a minus domination is the number of vertices that are assigned 1, (2) the minus-domination problem is polynomial for graphs of bounded rankwidth and for strongly chordal graphs, (3) it is NP-complete for split graphs, and (4) there is no fixed-parameter algorithm for minus-domination.

1. Introduction

The area of domination problems is affected by the recent fixed-parameter investigations (see, e.g., [1,2]). Let $G = (V, E)$ be a graph and let $f : V \to S$ be a function that assigns some integer from $S \subseteq \mathbb{Z}$ to every vertex of G. For a subset $W \subseteq V$ we write

$$f(W) = \sum_{x \in W} f(x).$$

The function f is a domination function if for every vertex x, $f(N[x]) > 0$, where $N[x] = \{x\} \cup N(x)$ is the closed neighborhood of x. The *weight* of f is defined as the value $f(V)$.

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http://dx.doi.org/10.1016/j.disopt.2016.04.002

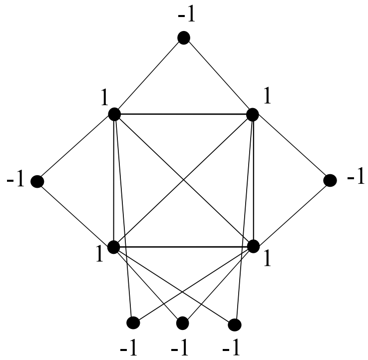

Fig. 1. A minus-domination function with negative weight.

In this manner, the ordinary domination problem is described by a domination function that assigns a value of $\{0, 1\}$ to each element of V. A signed domination function assigns a value of $\{-1, 1\}$ to each vertex x. The minimal weights over all dominating and signed dominating functions are denoted by $\gamma(G)$ and $\gamma_s(G)$, respectively. In this paper we look at the minus-domination problem.

Definition 1. Let $G = (V, E)$ be a graph. A function $f : V \to \{-1, 0, 1\}$ is a minus-domination function if $f(N[x]) > 0$ for every vertex x.

In the minus-domination problem one tries to minimize the weight of a minus-domination function. The minimal weight over all minus-domination functions is denoted as $\gamma^-(G)$. Notice that the weight may be negative. For example, consider a K_n with $n \geqslant 4$ and add one new vertex for every edge, adjacent to the endpoints of that edge. Assign a value 1 to every vertex of the K_n and assign a value -1 to each of the other vertices. This is a valid and optimal minus-domination function and its weight is $n - \binom{n}{2}$ (see Fig. 1 for an illustration).

The problem to determine the value of $\gamma^-(G)$ is NP-complete, even when restricted to bipartite graphs, chordal graphs and planar graphs with maximal degree four [3,4]. Damaschke shows that, unless $P = NP$, the value of γ^- cannot be approximated in polynomial time within a factor $1 + \epsilon$, for some $\epsilon > 0$, not even for graphs with maximum degree at most four [3, Theorem 7]. Sharp bounds for the minimum weight are obtained for some classes of graphs, e.g., graphs with $\Delta(G) \leqslant 3$ and 4 [3], trees [5], bipartite graphs [6,7], complete bipartite graphs [8], multipartite graphs [9], cubic graphs [10–12], regular graphs [13], and general graphs [14].

There are very few algorithmic results for solving the minus domination problem on some special graphs. As far as we know, there are only linear-time algorithms for solving the minus domination problem on trees [4], chain interval graphs [15], and strongly chordal graphs [16]. This motivates us to investigate the complexity of the minus-domination problem for some classes of perfect graphs including cographs, distance-hereditary graphs, strongly chordal graphs and split graphs. Moreover, the minus-domination problem is polynomial for graphs of bounded rankwidth and fixed-parameter tractable for d-degenerate graphs when parameterized by the size of the minus-dominating set and by d, where the size of a minus domination is the number of vertices that are assigned 1.

2. d-Degenerate graphs

Let $G = (V, E)$ be a graph and let $f : V \to S$ be a domination function. Following Zheng et al. [17], we define the *size* of f as the number of vertices $x \in V$ with $f(x) > 0$. We denote the size of a minus-dominating