



Progress on the description of identifying code polyhedra for some families of split graphs



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ABSTRACT

The identifying code problem is a newly emerging search problem, challenging both from a theoretical and a computational point of view, even for special graphs like bipartite graphs and split graphs. Hence, a typical line of attack for this problem is to determine minimum identifying codes of special graphs or to provide bounds for their size.

In this work we study the associated polyhedra for some families of split graphs: headless spiders and complete suns. We provide the according linear relaxations, discuss their combinatorial structure, and demonstrate how the associated polyhedra can be entirely described or polyhedral arguments can be applied to find minimum identifying codes for special split graphs. We discuss further lines of research in order to apply similar techniques to obtain strong lower bounds stemming from linear relaxations of the identifying code polyhedron, enhanced by suitable cutting planes to be used in a B&C framework.

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1. Introduction

Many practical applications can be stated as set covering problems. For instance, some newly emerging search problems like fault detection in networks, fire detection in buildings, or performing group tests, can be modeled as special variant of domination problems: so-called identifying codes in graphs [1].

Consider a graph $G = (V, E)$ and denote by $N[i] = \{i\} \cup N(i)$ the closed neighborhood of i . A subset $C \subseteq V$ is *dominating* (resp. *identifying*) if $N[i] \cap C$ are non-empty (resp. distinct) sets for all $i \in V$. An *identifying code* of G is a node subset which is dominating and identifying, see Fig. 1, and the *identifying code number* $\gamma^{ID}(G)$ of a graph G is the minimum cardinality of an identifying code of G .

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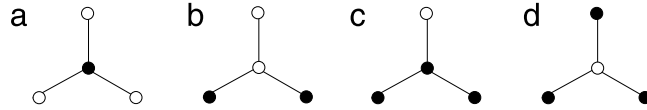


Fig. 1. A graph, where the subset of black nodes forms (a) a dominating (but not identifying) set, (b) an identifying (but not dominating) set, and (c), (d) minimum identifying codes.

Determining $\gamma^{ID}(G)$ is in general NP-complete [2]. It even remains hard for several graph classes where many other in general hard problems are easy to solve, including bipartite graphs [2], split graphs [3] and, thus, chordal graphs. From a graph theoretical point of view, the problem has been actively studied, see e.g. [4–7] for some recent papers in the area.

Polyhedral studies of the identifying code problem were initiated in [8,9]. For that, a suitable reformulation of the problem in terms of an integer linear program is in order. Determining a minimum identifying code in a graph $G = (V, E)$ can be formulated as set covering problem $\min \mathbf{1}^T x, M_{ID}(G) x \geq \mathbf{1}, x \in \{0, 1\}^{|V|}$ by:

$$\begin{aligned} \min \mathbf{1}^T x \\ x(N[j]) = \sum_{i \in N[j]} x_i &\geq 1 \quad \forall j \in V \quad (\text{domination}) \\ x(N[j] \triangle N[k]) = \sum_{i \in N[j] \triangle N[k]} x_i &\geq 1 \quad \forall j, k \in V, j \neq k \quad (\text{identification}) \\ x &\in \{0, 1\}^{|V|}. \end{aligned}$$

We call

$$M_{ID}(G) = \begin{pmatrix} N[G] \\ \triangle[G] \end{pmatrix}$$

the *identifying code matrix* of G , encoding the closed neighborhoods of the nodes of G ($N[G]$) and their symmetric differences ($\triangle[G]$), and define the *identifying code polyhedron* of G as

$$P_{ID}(G) = \text{conv}\{x \in \mathbb{Z}_+^{|V|} : M_{ID}(G) x \geq \mathbf{1}\}.$$

It is clear by construction that $\gamma^{ID}(G)$ equals the covering number

$$\tau(M_{ID}(G)) := \min\{\mathbf{1}^T x : x \in P_{ID}(G)\}.$$

In addition, a graph G has an identifying code or is *identifiable* if and only if $M_{ID}(G)$ has no zero-row. As $N[G]$ has clearly no zero-row, G is identifiable if and only if $\triangle[G]$ has no zero-row which is equivalent to the known condition that G is identifiable if and only if it has no true twins, i.e., nodes i, j with $N[i] = N[j]$, see [1].

As $M_{ID}(G)$ may contain rows which are equal to or dominated by other rows in $M_{ID}(G)$, we define the corresponding clutter matrix, the *identifying code clutter* $C_{ID}(G)$ of G , obtained by removing repeated or dominated rows from $M_{ID}(G)$. We clearly have that

$$P_{ID}(G) = \text{conv}\{x \in \mathbb{Z}_+^{|V|} : C_{ID}(G) x \geq \mathbf{1}\},$$

and obtain as a linear relaxation the *fractional identifying code polyhedron*

$$Q_{ID}(G) = \{x \in \mathbb{R}_+^{|V|} : C_{ID}(G) x \geq \mathbf{1}\}.$$

In [8,9] we characterized when $P_{ID}(G)$ is full-dimensional and which constraints of $Q_{ID}(G)$ define facets of $P_{ID}(G)$:

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