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Green's functions in orthotropic thermoelastic diffusion media

Rajneesh Kumar*, Vijay Chawla

Department of Mathematics, Kurukshetra University, Kurukshetra 136119, Haryana, India

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ABSTRACT

The aim of the present paper is to study the Green's function in orthotropic thermoelastic diffusion media. With this objective, firstly the two-dimensional general solution in orthotropic thermoelastic diffusion media is derived. On the basis of general solution, the Green's function for a steady point heat source in the interior of semi-infinite orthotropic thermoelastic diffusion material is constructed by four newly introduced harmonic functions. The components of displacement, stress, temperature distribution and mass concentration are expressed in terms of elementary functions. From the present investigation, a special case of interest is also deduced, to depict the effect of diffusion on components of stress and temperature distribution.

1. Introduction

Green's functions or fundamental solutions play an important role in both applied and theoretical studies on the physics of solids. Green's functions can be used to construct many analytical solutions solving boundary value problems of practical problems when boundary conditions are imposed. They are essential in boundary element method (BEM) as well as the study of cracks, defects and inclusion. Many researchers have investigated the Green's function for elastic solid in isotropic and anisotropic elastic media, notable among them are Freedholm [1], Lifshitz and Rezentsveig [2], Elliott [3], Kroner [4], Synge [5], Lejcek [6], Pan and Chou [7] and Pan and Yuan [8].

When thermal effects are considered, Sharma [9] investigated the fundamental solution for transversely isotropic thermoelastic material in an integral form. Chen et al. [10] derived the three dimensional general solution for transversely isotropic thermoelastic materials. Hou et al. [11,12] investigated the Green's function for two and three-dimensional problem for a steady point heat source in the interior of semi-infinite thermoelastic materials. Also, Hou et al. [13] investigated the two dimensional general solutions and fundamental solutions for orthotropic thermoelastic materials.

Diffusion is defined as the spontaneous movement of the particles from a high concentration region to the low concentration region and it occurs in response to a concentration region and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Today, thermal diffusion remains a practical process to separate isotopes of noble gases (e.g. xexon) and other light isotopes (e.g. carbon) for research purpose.

Nowacki [14–16] developed the theory of thermoelastic diffusion using coupled thermoelastic model. Nowacki [17] derived the basic equations for generalized thermoelastic diffusion. Sherief and Saleh [18] developed the generalized theory of thermoelastic diffusion with one relaxation time which allows finite speeds of propagation of waves. When diffusion effects are considered, Kumar and Kansal [19] derived the basic equations for generalized thermoelastic diffusion (G-L model) and discussed the Lamb waves. Also, Kumar and Chawla [20] investigated the Fundamental solution in orthotropic thermoelastic diffusion material. However, the important Green's function for two-dimensional problem for a steady point heat source in orthotropic thermoelastic diffusion material has not been discussed so far.

The Green's function for two-dimensional in orthotropic thermoelastic diffusive medium is investigated in this paper. Based on the two-dimensional general solution of orthotropic thermoelastic diffusion media, the Green's function for a steady point heat source in the interior of semi-infinite orthotropic thermoelastic diffusion material is constructed by four newly introduced harmonic functions. A special case of interest is also deduced to depict the effect of diffusion.

2. Basic equations

Following Sherief and Saleh [18] and Kumar and Kansal [19], the basic equations for homogenous anisotropic generalized

* Corresponding author.
E-mail addresses: rajneesh_kuk@rediffmail.com (R. Kumar), vijay.chawla@ymail.com (V. Chawla).

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thermoelastic diffusion solid in the absence of body forces, heat and mass diffusion sources are

(i) Constitutive relations:

$$\sigma_{ij}=c_{ijkm}\varepsilon_{km}+a_{ij}T+b_{ij}C, \tag{1}$$

(ii) Equations of motion:

$$c_{ijkm}\varepsilon_{km,j}+a_{ij}T_{,j}+b_{ij}C_{,j}=\rho\ddot{u}_i, \tag{2}$$

(iii) Equation of heat conduction:

$$\rho C_E\dot{T}+aT_0\dot{C}-a_{ij}T_0\dot{\varepsilon}_{ij}=K_{ij}T_{ij}, \tag{3}$$

(iv) Equation of mass diffusion:

$$-\alpha^*_{ij}b_{km}\varepsilon_{km,ij}-\alpha^*_{ij}bC_{,ij}+\alpha^*_{ij}aT_{,ij}=-\dot{C}, \tag{4}$$

here, $c_{ijkm}(=c_{kmij}=c_{jikm}=c_{ijmk})$ are the elastic parameter; $a_{ij}(=a_{ji})$, $b_{ij}(=b_{ji})$ are, respectively, the tensor of thermal and diffusion moduli. ρ is the density and C_E is the specific heat at constant strain, a, b are, respectively, coefficients describing the measure of thermoelastic diffusion effects and diffusion effects, T_0 is the reference temperature such that $|T/T_0|\ll 1$. $K_{ij}(=K_{ji})$, $\sigma_{ij}(=\sigma_{ji})$ and $\varepsilon_{ij}=(u_{i,j}+u_{j,i})/2$ denote the components of thermal conductivity, stress and strain tensor respectively. $T(x,y,z,t)$ is the temperature change from the reference temperature T_0 and C is the mass concentration. u_i are components of displacement vector. $\alpha^*_{ij}(=\alpha^*_{ji})$ are diffusion parameters.

In the above equations symbol (",") followed by a suffix denotes differentiation with respect to spatial coordinate and a superposed dot (".") denotes the derivative with respect to time.

3. Formulation of the problem

We consider homogenous orthotropic thermoelastic diffusion medium. Let us take $Oxyz$ as the frame of reference in Cartesian coordinates, the origin O being any point on the plane boundary.

For two-dimensional problem, we assume the displacement vector, temperature change and mass concentration are, respectively, of the form

$$\vec{u}=(u,0,w),\quad T(x,z,t),\quad C(x,z,t). \tag{5}$$

Moreover, we are discussing steady problem

$$\frac{\partial u}{\partial t}=\frac{\partial w}{\partial t}=\frac{\partial C}{\partial t}=\frac{\partial T}{\partial t}=0. \tag{6}$$

We define the dimensionless quantities as

$$x'=\frac{\omega_1^* x_i}{v_1},\quad z'=\frac{\omega_1^* z}{v_1},\quad u'=\frac{\omega_1^* u}{v_1},\quad w'=\frac{\omega_1^* w}{v_1},$$
$$T'=\frac{a_1T}{c_{11}},\quad C'=\frac{b_1C}{c_{11}},\quad \sigma'_{ij}=\frac{\sigma_{ij}}{a_1T_0},\quad H'=\frac{a_1v_1}{c_{11}K_1\omega_1^*}H,$$

where

$$v_1^2=b_1,\quad \omega_1^*=\frac{ac_{11}}{K_1}, \tag{7}$$

and b_1 is the tensor of diffusion moduli and K_1 is the component of thermal conductivity.

Eqs. (2)–(4) for orthotropic materials, with the aid of (5)–(7), after suppressing the primes, yield

$$\left(\frac{\partial^2}{\partial x^2}+\delta_1\frac{\partial^2}{\partial z^2}\right)u+\left(\delta_2\frac{\partial^2}{\partial x\partial z}\right)w-\left(\frac{\partial}{\partial x}\right)T-\left(\frac{\partial}{\partial x}\right)C=0, \tag{8}$$

$$\left(\delta_2\frac{\partial^2}{\partial x\partial z}\right)u+\left(\delta_1\frac{\partial^2}{\partial x^2}+\delta_3\frac{\partial^2}{\partial z^2}\right)w-\varepsilon_1\left(\frac{\partial}{\partial z}\right)T-\varepsilon_2\left(\frac{\partial}{\partial z}\right)C=0, \tag{9}$$

$$\left(\frac{\partial^2}{\partial x^2}\right)T+\varepsilon_3\left(\frac{\partial^2}{\partial z^2}\right)T=0, \tag{10}$$

$$\frac{\partial}{\partial x}\left(q_1^*\frac{\partial^2}{\partial x^2}+q_7^*\frac{\partial^2}{\partial z^2}\right)u+\frac{\partial}{\partial z}\left(q_8^*\frac{\partial^2}{\partial x^2}+q_2^*\frac{\partial^2}{\partial z^2}\right)w$$
$$+\left(q_3^*\frac{\partial^2}{\partial x^2}+q_4^*\frac{\partial^2}{\partial z^2}\right)T-\left(q_5^*\frac{\partial^2}{\partial x^2}+q_6^*\frac{\partial^2}{\partial z^2}\right)C=0, \tag{11}$$

where

$$\delta_1=\frac{c_{44}}{c_{11}},\delta_2=\frac{c_{13}+c_{44}}{c_{11}},\delta_3=\frac{c_{33}}{c_{11}},\varepsilon_1=\frac{a_3}{a_1},\varepsilon_2=\frac{b_3}{b_1}.$$

$$q_1^*=\frac{\alpha_1^*\omega_1^*b_1}{c_{11}},\quad q_2^*=\frac{\alpha_3^*\omega_1^*b_3}{c_{11}},\quad q_5^*=\frac{\alpha_1^*\omega_1^*b}{b_1},\quad q_6^*=\frac{\alpha_3^*\omega_1^*b}{b_1},$$
$$q_3^*=\frac{\alpha_1^*\omega_1^*a}{a_1},\quad q_4^*=\frac{\alpha_3^*\omega_1^*a}{a_1},\quad q_7^*=\frac{\alpha_3^*\omega_1^*b_1}{c_{11}},\quad q_8^*=\frac{\alpha_1^*\omega_1^*b_3}{c_{11}}.$$

By virtue of the parallel method of Chen et al. [10], the general solution as obtained by Kumar and Chawla [20] gives

$$u=\sum_{j=1}^4\frac{\partial\psi_j}{\partial x},\quad w=\sum_{j=1}^4 s_jP_{1j}\frac{\partial\psi_j}{\partial z_j},\quad C=\sum_{j=1}^4P_{2j}\frac{\partial^2\psi_j}{\partial z_j^2},\quad T=\sum_{j=1}^4P_{34}\frac{\partial^2\psi_4}{\partial z_4^2},$$
$$\sigma_{xx}=-\sum_{j=1}^4s_j^2w_{1j}\frac{\partial^2\psi_j}{\partial z_j^2},\quad \sigma_{zz}=\sum_{j=1}^4w_{1j}\frac{\partial^2\psi_j}{\partial z_j^2},\quad \sigma_{zx}=\sum_{j=1}^4s_jw_{1j}\frac{\partial^2\psi_j}{\partial x\partial z_j}, \tag{12}$$

where ψ_j satisfies the harmonic equations

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z_j^2}\right)\psi_j=0,\quad j=1,2,3,4 \tag{13}$$

and

$$P_{1j}=p_{2j}/p_{1j},\quad P_{2j}=p_{3j}/p_{1j},\quad P_{34}=p_{44}/p_{14}, \tag{14a}$$

$$w_{1j}=\frac{f_1-f_2s_j^2P_{1j}+f_1P_{3j}+f_1P_{2j}}{s_j^2}=h_4(1+P_{1j})=-f_2+h_1s_j^2P_{1j}-h_2P_{3j}-h_3P_{3j}, \tag{14b}$$

$f_1=c_{11}/a_1T_0$, $f_2=c_{13}/a_1T_0$, $h_1=c_{33}/a_1T_0$, $h_2=a_3f_1/a_1$, $h_3=b_3 f_1/b_1$, $h_4=c_{44}/a_1T_0$, $z_j=s_jz$, $s_4=\sqrt{K_1/K_3}$, and $s_j(j=1,2,3)$ are three roots (with positive real part) of the algebraic equation (Eq. (19) of Kumar and Chawla [20]).

4. Green's function for a point heat source in the interior of semi-infinite orthotropic thermoelastic diffusion materials

As shown in Fig. 1 we consider an orthotropic semi-infinite thermoelastic diffusion plane $z\geq 0$. A point heat source H is applied at the point $(0,h)$ in two dimensional Cartesian coordinate (x,z) and the surface $z=0$ is free, impermeable boundary and thermally insulated. The general solution given by Eq. (12) is derived in this section.

The boundary conditions on the surface $z=0$ are

(i) Mechanical condition:

$$\sigma_{zz}=\sigma_{zx}=0, \tag{15a}$$

(ii) Concentration condition:

$$\frac{\partial C}{\partial z}=0, \tag{15b}$$