Operations Research Letters 45 (2017) 461–466

Contents lists available at ScienceDirect

Operations Research Letters

journal homepage: www.elsevier.com/locate/orl

The Stable Tournament Problem: Matching sports schedules with preferences

Mario Guajardo *, Kurt Jörnsten

Department of Business and Management Science, NHH Norwegian School of Economics, N-5045 Bergen, Norway

ARTICLE INFO

Article history:
Received 19 August 2016
Received in revised form 14 July 2017
Accepted 14 July 2017
Available online 27 July 2017

Keywords:
Sports scheduling
Stable matching
Preferences
Integer programming

ABSTRACT

By combining sports scheduling with stable matching literature, this article defines the Stable Tournament Problem, which attempts to find a schedule of games that is stable with respect to teams' preferences. In every round of a stable schedule, no pair of teams who have not played against each other has to play against less preferable opponents. An integer linear model is formulated and results are provided for instances with random preferences and preferences structured according to the strength of the teams.

1. Introduction

Although operations research techniques have allowed to improve the way sports competitions are scheduled, it is not rare that teams' coaches, players or fans complain about the schedules. In the Chilean football league, for example, [3,4] have used integer linear programming to schedule all tournaments of the First Division since 2005 and of the second Division since 2007. While this has allowed to effectively capture a series of criteria over the years and to provide many benefits to the leagues, it seems almost impossible to fulfill the expectations of all the involved actors. For example, when the schedule of the 2013 First Division season was released, Víctor Hugo Castañeda, former coach of Everton, told the press that the schedule was very disadvantageous for his team. The tournament in question was a single round robin, thus unavoidably some of the opponents had to be played at home while others away. Castañeda's argument was that Everton was scheduled to play as visitor against four of the weakest teams, with whom he expected to fight for not being relegated to the Second Division. Ironically, it ended up being one of the best seasons in the last years for Everton, who reached a very safe sixth place in the standings, far above the teams that were relegated. Also in football, [7] has used mixed integer linear programming to schedule the Belgian Jupiler League since 2006. Although the application received a very positive response, negative comments from the clubs seem unavoidable. They quote Guy Mangelschots, former coach of Sint-Truiden, who puts it very eloquently: "When after five games you look at the points you have, only then you can say whether the schedule is good or bad".

It is arguable whether the judgment of the coaches is well founded or they simply point to a tough schedule to hedge against bad performances of their teams. What seems unarguably hard is to create a schedule that satisfies all parties involved in a sport competition. If each team would state its own preferred schedule, could we find a schedule such that in every round there is no pair of teams who could be better off playing against each other instead of against the teams they are scheduled to play with? This motivates us to formulate a new problem that we call the *Stable Tournament Problem* (STP). The purpose of this article is first to formally define this problem. Second, we formulate an integer linear model that finds a solution to this problem, whenever this is feasible. Third, we provide numerical results for a number of instances. In some of these, the preferences of the teams are created randomly. In others, the preferences follow certain structures that we find interesting to study in sports competitions.

2. Problem definition

Let I be a set of n teams and K a set of $n-1$ rounds, where n is an even number. In a *single round robin tournament*, every team must play once against every other team. A tournament is *compact* if every team must play one game on every round. For every $i \in I$, define a bijective function $G_i : I \setminus \{i\} \to K$ representing i's preferred schedule. That is, $G_i(j)$ indicates the round on which team i wants to play against team j. Note the bijection implies a complete preference list, in which every team orders the other teams strictly from earliest to latest preferred opponent. If team i

* Corresponding author.
E-mail addresses: mario.guajardo@nhh.no (M. Guajardo), kurt.jornsten@nhh.no (K. Jörnsten).

http://dx.doi.org/10.1016/j.orl.2017.07.004
0167-6377/

prefers to play against team h on an earlier round than against team j (that is, $G_i(h) < G_i(j)$), we write $h <_i j$.

We define a schedule S as a set of *games* or tuples (i, j, k) indicating that team i plays against team j ($j \neq i$) on round k. For a compact single round robin tournament, a schedule must contain one tuple of the type $(i, \cdot, k)$ and one tuple of the type $(i, j, \cdot)$ for all $k \in K, i \in I, j \in I : j \neq i$.

Let i, j, h, p be any four different teams, and k_1 and k_2 any two different rounds such that $k_1 < k_2$. Let (i, j, k_1), (h, p, k_1), (i, h, k_2) be games scheduled in S. If either $j <_i h$ or $p <_h i$ or both, we say that S is *stable*. In other words, a schedule is stable if in every round there is no pair of teams who have not yet played against each other and they prefer to play against each other earlier than against the respective opponents they have been scheduled to play against in this round.

Table 1 illustrates this stability concept for a tournament of 4 teams. While the first schedule is stable, the second one is not because in round 1 team 1 and team 2 would prefer to play against each other instead of playing against teams 3 and 4, respectively.

Given the preferences of the teams, the STP consists of finding a stable schedule. In particular, the aim is to find a stable schedule which is as similar as possible to the preferred schedule of the teams. As measure of similarity, we use the sum over all teams of the absolute value of the difference between the round a team wants to play against every other team and the actual round they are scheduled to play.

Besides sports competitions, the STP relates to recent scheduling literature that takes into account the preferences of the agents affected by the schedules. For example, [1] develops preference-based approaches for staff scheduling, and [8] for scheduling groups of students at universities. The STP also relates to the vast stream of literature on stable matching originated by [6]. There are many variants of stable matching problems. The closest ones to the STP are the *stable fixtures problem* (SFP), introduced by [10], and the *stable multiple activities problem* (SMAP), introduced by [2]. The SFP generalizes the classical *stable roommates problem* [6,9] by allowing polygamy. The polygamy possibility for a team is characterized by its *capacity*, which indicates the maximum number of matches it can be assigned. The SMAP is an even more general variant, in which, besides polygamy, a team is allowed to get involved in different activities with the other teams. To the best of our knowledge, the single round robin and compactness conditions (which are inherently linked to sports tournaments divided in rounds) together with a measure of similarity, distinguish the STP as a novel variant in the literature. It resembles the SFP or a single-activity instance of the SMAP in which the capacity of all teams is equal to $n - 1$ but instead of interpreting this capacity as a target number of matches (which may or may not be played), the STP requires all teams to play exactly once against each other. In addition, the STP incorporates the temporal notion of round, which is absent in both the SFP and the SMAP. This concept follows the usual feature of sports competitions that limit teams to play exactly one game per round.

3. Integer linear formulation

In the following, we formulate an integer linear model for the STP. The decision variables are: x_{ijk} equal to 1 if team i plays against team j in round k, 0 otherwise ($i, j \in I : i \neq j$); y_{ijk} equal to 1 if in round k team i plays against a team h such that $G_i(h) < G_i(j)$ and j plays against a team p such that $G_j(p) < G_j(i)$, and zero otherwise ($i, j \in I : i \neq j$); and Δ_{ij} is the absolute value difference between the round in which i plays against j and the round i wants to play against j ($i, j \in I : i \neq j$). The objective function and constraints are given below.

$$\min f = \sum_{i \in I} \sum_{\substack{j \in I \\ j \neq i}} \Delta_{ij} \tag{1}$$

s.t.

$$\sum_{k \in K} x_{ijk} = 1 \qquad \forall i \in I, j \in I : i \neq j \tag{2}$$

$$\sum_{\substack{j \in I \\ j \neq i}} x_{ijk} = 1 \qquad \forall i \in I, k \in K \tag{3}$$

$$\sum_{\substack{\bar{k} \in K \\ \bar{k} \leq k}} \sum_{\substack{h \in I \\ h <_i j}} x_{ih\bar{k}} + \sum_{\substack{\bar{k} \in K \\ \bar{k} \leq k}} \sum_{\substack{h \in I \\ h <_j i}} x_{jh\bar{k}} - \sum_{\substack{\bar{k} \in K \\ \bar{k} \leq k}} y_{ij\bar{k}} + \sum_{\substack{\bar{k} \in K \\ \bar{k} \leq k}} (k - \bar{k} + 1) x_{ij\bar{k}} \geq k \quad \forall k \in K, i \in I, j \in I : i \neq j \tag{4}$$

$$y_{ijk} \leq \sum_{\substack{h \in I \\ h <_i j}} x_{ihk} \quad \forall k \in K, i \in I, j \in I : i \neq j \tag{5}$$

$$y_{ijk} \leq \sum_{\substack{h \in I \\ h <_j i}} x_{jhk} \quad \forall k \in K, i \in I, j \in I : i \neq j \tag{6}$$

$$\sum_{\substack{h \in I \\ h <_i j}} x_{ihk} + \sum_{\substack{h \in I \\ h <_j i}} x_{jhk} \leq 1 + y_{ijk} \quad \forall k \in K, i \in I, j \in I : i \neq j \tag{7}$$

$$\Delta_{ij} \geq \sum_{k \in K} k x_{ijk} - G_i(j) \qquad \forall i \in I, j \in I : i \neq j \tag{8}$$

$$\Delta_{ij} \geq G_i(j) - \sum_{k \in K} k x_{ijk} \qquad \forall i \in I, j \in I : i \neq j \tag{9}$$

$$x_{ijk} = x_{jik} \qquad \forall k \in K, i \in I, j \in I : i \neq j \tag{10}$$

$$x_{ijk}, y_{ijk} \in \{0, 1\}, \Delta_{ij} \geq 0 \quad \forall k \in K, i \in I, j \in I : i \neq j. \tag{11}$$

Objective function (1) minimizes the deviation of the schedule with respect to the preferences of the teams. Constraints (2) state that every team must play once against every other team. Constraints (3) state that every team plays one match every round. Constraints (4) model stability for all the rounds of the tournament. They resemble the stable matching constraints introduced by [13] for the classical *stable marriage problem* [6]. Note, however, instead of a single stage of matches, a sport tournament is divided into rounds and the stability constraints are required for each round k and each pair of teams i and j. For a schedule to be stable, it must hold that if i and j have not played before round k, in every previous round at least one of them played against a more preferable opponent. This is captured by the first two terms on the left-hand side of constraints (4). The first one counts the number of rounds previous to k in which i has played with opponents more preferable than j. The second term counts the number of rounds previous to k in which j has played with opponents more preferable than i. Since in a same round both i and j could have played against more preferable opponents, we need to subtract these rounds from the sum of the previous two terms to avoid double counting. This is done by the third term on the left-hand side. The fourth term is zero if teams i and j have not played against each other before round k, otherwise it counts the number of rounds since they played until the end of round k. Constraints (5)–(7) are logical relationships between variables x and y. Constraints (8)–(9) are logical relationships between variables x and Δ. Constraints (10) are logical relationships of symmetry (they can be dropped if x_{ijk} is defined over ordered pairs (i, j), but we include them here to