Operations Research Letters 44 (2016) 737–741

Contents lists available at ScienceDirect

Operations Research Letters

journal homepage: www.elsevier.com/locate/orl

Opaque selling in congested systems

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ARTICLE INFO

Article history:
Received 15 March 2016
Received in revised form
10 September 2016
Accepted 10 September 2016
Available online 17 September 2016

Keywords:
Opaque selling
Uniform pricing
Congestion

ABSTRACT

Opaque selling, whereby the firm hides some product attribute until payment is completed, is proved effective in maximizing profit if used with price discrimination. We study opaque selling in a congestion-susceptible environment, which has received little attention in the literature, and show its advantages without price discrimination in two particular cases.

1. Introduction

Opaque selling is a pricing strategy that has been employed in various industries. The implementation of this strategy is essentially to offer the customers a lottery of getting one of the differentiated products at a uniform price. As a result, the information on certain product attributes is withheld from the customers at the time of purchase. Airline companies hiding flight time, classes, or even destinations to the customers who buy their opaque services appears to be the first and best-known example [6]. In addition, retailing firms have also tried opaque selling for better revenue performance [2].

Recently, the opaque selling strategy has stimulated a fast-growing body of research. Scholars have been trying to find out under what condition, and why, opaque selling is attractive to firms. One of the important results, discovered by several papers (e.g. [6,2]), is that opaque selling captures a larger market and, if used along with regular selling strategy, increases the profit for the firm. Specifically, these papers study a monopolist firm facing Hotelling line type of customers. The firm can either sell regular (transparent) services or offer an opaque product, or a combination of both. While selling regular products only captures the customers at the two ends, offering the opaque product alone will eliminate the horizontal differentiation and thus expand the market coverage. However, due to the cannibalization effect, the total profit will shrink. One remedy, as proposed by these papers, is to offer both regular and opaque products to the customers at different prices. In this way, the firm takes advantage of *market segmentation* and *price discrimination*, and consequently captures a larger market with higher profit.

The setting in the previous papers is a commonly used one, and the result is fundamental. However, we note two limitations. First, the extant literature has studied opaque selling in service industries, but never considers congestion effect. Nevertheless, customers who seek to procure services may have to wait in the systems, which affects the firm's strategy. Hence, the congestion-susceptible settings where queues are unavoidable and costly become plausible and require investigation. In particular, the previous result that opaque selling expands the market should be re-examined. Second, although price discrimination is advocated for the success of opaque selling, its "dark" side has been well-documented [8]. Two common arguments against price discrimination are as follows. (1) People prefer simpler rules. Differentiated pricing based on customers' characteristics is clearly more complex to assess than simple uniform pricing [4]. (2) "Equal-price" conveys more sense of fairness to customers whereas different prices may induce a perception of unfairness [9]. Therefore, it is interesting to ask whether opaque selling can still enhance profit without price discrimination.

Based on the above discussions, this paper attempts to make two extensions on the previous works. Our baseline model, which is similar to Jiang [6], looks at a service firm with two identical servers and customers residing on a unit line. We further assume that price discrimination is not feasible in our setting, and only focus on *pure* opaque selling, where the firm just offers the opaque service at a uniform price to all customers. Of course, the firm can just sell regular (transparent) services.

The primary research question is how pure opaque selling compares to regular selling in terms of market coverage and profit. We find that, contrary to the known result, pure opaque selling

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http://dx.doi.org/10.1016/j.orl.2016.09.005
0167-6377/

does not always capture more market share. Our results reveal the significance of the congestion term, which is not seen in the previous works. Moreover, we identify two situations where pure opaque selling leads to higher profit. Hence, this paper advances the literature to establish positive arguments concerning the profit advantages of opaque selling without price discrimination.

2. Baseline model and analysis

Consider a firm (service provider) with two identical servers (indexed 1 and 2) located at each end of a Hotelling line ([0, 1]). Potential customers reside uniformly on the line. They have identical valuation on the two servers, which is set to be 1. Customers are subject to transportation cost t per unit distance ($0 < t < 1$) if they decide to procure service. Specifically, valuations on the two servers by the customer located at $x \in [0, 1]$ are $v_1(x) = 1 - tx$ and $v_2(x) = 1 - t(1 - x)$, respectively.

We depart from the existing literature as we model the service at each server by an M/M/1 queue. Hence, we follow the research on equilibrium behaviors in queueing systems [5], and assume that the queue length is not observable to customers upon arrival. The congestion cost due to waiting lines is linear (c per customer per unit time) and is incurred to the firm. Then, for any price p set by the firm, all customers with valuation higher or equal to p will join the system. To give an example, consider a make-to-order firm that owns two factories at the ends of a linear city; its objective is to price the order made to each factory in order to maximize profit.

We first assume that the firm employs regular selling strategy. Due to symmetry, the firm will charge the same price to all customers, and thus we just consider one side of the market, e.g. server 1. Consider the customer whose valuation on procuring service from server 1 equals to the price p. Suppose that this customer locates at x ($0 \leq x \leq 1/2$) on the Hotelling line. Therefore, $p = v_1(x)$ and the total market coverage $M_r = 2x \in [0, 1]$. In addition, the expected wait time for each customer at server 1 is $W = (\mu - x)^{-1}$, where $\mu > 1$ is the common capacity and is assumed to be ample and exogenous. As a result, the firm can equivalently maximize the total profit over the market coverage:

$$\pi_r^* := \max_{0 \leq x \leq 1/2} f(x), \quad \text{where } f(x) = 2\left(x v_1(x) - \frac{cx}{\mu - x}\right). \tag{1}$$

The firm may alternatively offer an opaque service (with a uniform price p_o) that does not specify which server the order is sent to before the payment is made. Instead, only the probabilities of where the service transpires is provided. Furthermore, we assume that a paid customer is sent immediately to one of the servers and wait (if necessary) in two separate queues. In other words, the firm promises early commitment [3] and randomly routes customers as soon as purchase is made. This assumption is reasonable when, for example, the last-minute transportation is prohibitively costly. Let β be the probability that the order is sent to factory 1; we set $\beta = 1/2$ in this paper due to the identical servers assumption. Hence, the customers' expected valuation towards this opaque service is $v_o(x) = v_1(x)/2 + v_2(x)/2 = 1 - t/2$, which is independent of the customer location on the Hotelling line; let $v_o = 1 - t/2$. Following the method used in [5], suppose that each customer uses a symmetric random strategy and procures the opaque service with probability $\alpha \in [0, 1]$. Then, the market coverage under opaque selling strategy is $M_o = \alpha$. Moreover, let $y = \alpha/2 \in [0, 1/2]$ be the customers volume for each server. Thus, every customer's expected waiting time is $W_o = (\mu - y)^{-1}$. Finally, the firm will fully exploit the customers' surplus, and thus equivalently maximizes the total profit over y:

$$\pi_o^* := \max_{0 \leq y \leq 1/2} g(y), \quad \text{where } g(y) = 2y v_o - \frac{2cy}{\mu - y}. \tag{2}$$

We now compare these two pricing strategies in terms of the optimal profit and market coverage.

Proposition 2.1. *Let M_r^* and M_o^* be the optimal market coverage under regular selling and opaque selling, respectively. Suppose that $v_o > c/\mu$, then*

(1) $\pi_r^* \geq \pi_o^*$; and $\pi_r^* = \pi_o^*$ if and only if $M_r^* = M_o^* = 1$.
(2) $M_o^* < M_r^* < 1/2$ if and only if $\frac{v_o}{c} < \frac{\mu}{(\mu - 1/4)^2}$.

Proof. Let x^* and y^* be optimal solutions to problems (1) and (2), respectively. Then,

$$\pi_o^* = 2y^*\left(1 - t/2 - \frac{c}{\mu - y^*}\right) \leq 2y^*\left(1 - ty^* - \frac{c}{\mu - y^*}\right) = f(y^*) \leq \pi_r^*,$$

because $0 \leq y^* \leq 1/2$. Note that the functions f and g are strictly concave with positive optimizer:

$$\frac{1}{2}f'(x) = 1 - 2tx - \frac{c\mu}{(\mu - x)^2}, \qquad f'(0) > 0,$$
$$\frac{1}{2}f''(x) = -2t - \frac{2c\mu}{(\mu - x)^3} < 0;$$
$$\frac{1}{2}g'(y) = 1 - t/2 - \frac{c\mu}{(\mu - y)^2}, \qquad g'(0) > 0,$$
$$\frac{1}{2}g''(y) = -\frac{2c\mu}{(\mu - y)^3} < 0.$$

If $M_r^* = M_o^* = 1$, i.e. $x^* = y^* = 1/2$, then $\pi_o^* = g(1/2) = f(1/2) = \pi_r^*$. To prove the reverse, suppose that $\pi_o^* = \pi_r^*$. If $0 \leq y^* < 1/2$, then the strict inequality $g(y^*) = f(y^*) - t(1 - 2y^*) < f(y^*) \leq f(x^*)$ is a contradiction. Hence, $y^* = 1/2$, and $f(x^*) = \pi_o^* = g(1/2) = f(1/2)$. In addition, the strictly concave function $f(x)$ has a unique maximizer over $[0, 1/2]$, resulting in $x^* = 1/2$. Therefore, $M_r^* = M_o^* = 1$.

To show (2), assume that $1/\mu < v_o/c < \frac{\mu}{(\mu - 1/4)^2}$. Then, $g'(0) > 0$ and $g'(1/2) < 0$, which means that there exists a unique maximizer $y^* \in (0, 1/2)$ such that $g'(y^*) = 0$. Solving for it yields $y^* = \mu - \sqrt{\frac{c\mu}{v_o}}$. By direct substitution, we have

$$\frac{1}{2}f'(1/4) = 1 - \frac{t}{2} - \frac{c\mu}{(\mu - 1/4)^2} < 0,$$

and

$$\frac{1}{2}f'(y^*) = 1 - 2ty^* - v_o = 2t\left(\frac{1}{4} - \left(\mu - \sqrt{\frac{c\mu}{v_o}}\right)\right) > 0.$$

Because $f(x)$ is strictly concave on $[0, 1/2]$, we deduce that its optimum x^* satisfies $y^* < x^* < 1/4$, which gives $M_o^* < M_r^* < 1/2$. Since all steps of the above deduction are reversible, we are done. □

Proposition 2.1(1) echoes with the result in [6] that the pure opaque selling is weakly dominated by regular selling. Result (2), however, is in contrast to the previous findings in literature. It shows the importance of the congestion term in the problem. Indeed, the condition in Proposition 2.1(2) indicates that, when the congestion cost is too large compared to service valuation, the firm does not want the system to be crowded, and therefore intentionally cover a smaller portion of the market.

Finally, we make a remark on the possible use of quadratic form of customer's valuation. If $v_1(x) = 1 - tx^2$ and $v_2(x) = 1 - t(1-x)^2$, then since $x \in [0, 1]$, the customers' valuation of the services is actually larger than the linear case. Besides, the valuation under regular selling increases more than that under opaque selling, and therefore the optimal profit generated by opaque selling is still