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A new infinite boundary element formulation combined to an alternative multi-region technique

Dimas Betioli Ribeiro [a,*], João Batista de Paiva [b]

[a] Polytechnic School of the University of São Paulo, Av. Prof. Almeida Prado tv. 2, n. 83, 05508-070 São Paulo, SP, Brazil
[b] São Carlos Engineering School of the University of São Paulo, Structural Engineering Department, Av. Trabalhador Sãocarlense, 400, 13566-590 São Carlos, SP, Brazil

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ABSTRACT

The main objective of this work is to obtain an efficient three-dimensional boundary element (BE) formulation for the simulation of layered solids. This formulation is obtained by combining an alternative multi-region technique with an infinite boundary element (IBE) formulation. It is demonstrated that such a combination is straightforward and can be easily programmed. Kelvin fundamental solutions are employed, considering the static analysis of isotropic and linear-elastic domains. Establishing relations between the displacement fundamental solutions of the different domains, the alternative technique used in this paper allows analyzing all domains as a single solid, not requiring equilibrium or compatibility equations. It was shown in a previous paper that this approach leads to a smaller system of equations when compared to the usual multi-region technique and the results obtained are more accurate. The two-dimensionally mapped infinite boundary element (IBE) formulation here used is based on a triangular BE with linear shape functions. One advantage of this formulation over quadratic or higher order elements is that no additional degrees of freedom are added to the original BE mesh by the presence of the IBEs. Thus, the IBEs allow the mesh to be reduced without compromising the accuracy of the result. The use of IBEs improves the advantages of the alternative multi-region technique, contributing for the low computational cost and allowing a considerable mesh reduction. Furthermore, the results show good agreement with the ones given in other works, confirming the accuracy of the presented formulation.

1. Introduction

Considering specifically infinite multi-domain models, many options are available in the literature and each one of them implies advantages and disadvantages. However, depending on the problem to be solved, one technique may become more attractive than the others.

In most cases, a numerical approach may be employed. The finite element method (FEM) is popular [1], however, it has some disadvantages when compared to other options such as the boundary element method (BEM) [2]. The FEM requires the discretization of the infinite domain, implying on a high number of elements and leading to a large and sometimes impracticable processing time. To reduce these inconveniences, some authors use infinite elements together with finite elements, such as [3].

It becomes more viable to solve these problems with the BEM, once only the boundary of the domains requires discretization. This allows reducing the problem dimension, implying on less processing time. This advantage is explored in several works [4] and more developments are making the BEM even more attractive to future applications [5]. The classical way to consider domains in contact with the BEM, which is described in detail in Ref. [6], is based on imposing equilibrium and compatibility conditions for all interface points between every pair of domains in contact. As pointed out in Ref. [7], these impositions may cause inaccuracies and numerous blocks of zeros are generated at the final system of equations. To avoid these disadvantages, Ref. [7] presents an alternative multi-region BEM technique for three-dimensional elastic problems, which does not require equilibrium nor compatibility conditions along the interfaces. This technique is also employed for two-dimensional elastic and potential problems in [8] and for bending plate analysis in [9] and [10]. Considering a constant Poisson ratio, it is possible to establish relations between the displacement fundamental solutions and to analyze all subdomains as a single solid. Thus, a better continuity between domains in contact is guaranteed and therefore the result accuracy is improved. In addition to that, no blocks of zeros are present at the final system of equations, which is reduced. Thus, better results are obtained in less processing time. By testing this

* Corresponding author. Tel.: +55 11 3091 5145; fax: +55 11 3091 5181.
E-mail addresses: dimas.ribeiro@usp.br, dimasbetioli@gmail.com (D.B. Ribeiro), paiva@sc.usp.br (J.B. de Paiva).

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formulation in heterogeneous domain problems with different Poisson ratios, the authors concluded that the error introduced by an average Poisson ratio consideration may be considered of little relevance for displacement calculation. In the end, it is viable to employ this formulation in more general engineering problems.

Another way to improve the BEM performance is by using infinite boundary elements (IBEs). The first reference to an IBE was [11], in which the shape functions of an origin BE are multiplied by special decay functions. Another type of IBE may be obtained by using mapped functions to relate the local system of coordinates to the global one, as originally shown in Ref. [12]. In those studies that make use of two-dimensional IBEs, such as performed in Ref. [13], it may be noted that they are generally based on quadrilateral BEs. An alternative to this type of IBE is given in Ref. [14], which presented a mapped IBE based on a triangular BE with linear shape functions. One advantage of this approach over quadratic or higher order elements is that no additional degrees of freedom are added to the original BE mesh by the presence of the IBEs.

The aim of this work is to combine the BE multi-region technique presented in Ref. [7] to the IBE formulation presented in Ref. [14], obtaining a new and more efficient multi-region BE formulation for layered domain simulation. The addition of IBE-type elements do not interfere with the alternative multi-region technique, so the combined formulation can be easily obtained and programmed. The domain is modeled with variable elasticity modulus and a constant Poisson ratio, as described by [15]. The results obtained are consistent with those of other authors, confirming the accuracy of the presented approach. In addition to that, the use of IBEs contributed for the low computational cost, allowing a considerable mesh reduction.

It is important to notice that the numerical versatility of the BEM is fully maintained, allowing an efficient simulation of a wide variety of problems. Furthermore, the main reason for combining the BE multi-region technique to the IBE formulation is that their advantages are united with no drawbacks. To their best knowledge, the authors are not aware of other numerical tools in the literature capable of simulating three-dimensional layered infinite domains with less degrees of freedom. Therefore, it may be considered a powerful alternative for other authors and a relevant contribution of this paper. In future works, we intend to apply this formulation in soil–structure interaction problems.

2. Boundary element formulation

The equilibrium of a solid body can be represented by a boundary integral equation called the Somigliana identity, which for homogeneous, isotropic and linear-elastic domains is

$$c_{ij}(y)u_j(y)+\int_{\Gamma} p^*_{ij}(x,y)u_j(x)\,d\Gamma(x)=\int_{\Gamma} u^*_{ij}(x,y)p_j(x)\,d\Gamma(x) \tag{1}$$

Eq. (1) is written for a source point y at the boundary, where the displacement is $u_j(y)$. The constant c_{ij} depends on the Poisson ratio and the boundary geometry at y, as pointed out in Ref. [13]. The field point x goes through the whole boundary Γ, where displacements are $u_j(x)$ and tractions are $p_j(x)$. The integral kernels $u^*_{ij}(x,y)$ and $p^*_{ij}(x,y)$ are Kelvin three-dimensional fundamental solutions for displacements and tractions, respectively. Kernel $u^*_{ij}(x,y)$ has order $1/r$ and kernel $p^*_{ij}(x,y)$ order $1/r^2$, where $r=|x-y|$, so the integrals have singularity problems when x approaches y. Therefore the stronger singular integral, over the traction kernel, has to be defined in terms of a Cauchy principal value (CPV).

To solve Eq. (1) numerically, the boundary is divided into regions within which displacements and tractions are approximated by known shape functions. Here these regions are of two types, finite boundary elements (BEs) and infinite boundary elements (IBEs). The BEs employed are triangular, as shown in Fig. 1 with the local system of coordinates, $\xi_1\xi_2$, and the local node numbering. The following approximations are used for this BE:

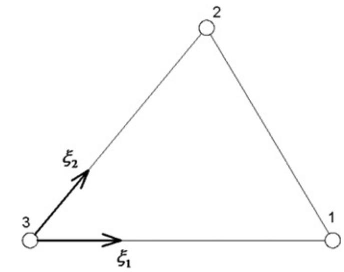

Fig. 1. Triangular boundary element.

$$u_j=\sum_{k=1}^{3}N^k u_j^k,\quad p_j=\sum_{k=1}^{3}N^k p_j^k \tag{2}$$

Eq. (2) relates the boundary values u_j and p_j to the nodal values of the BE. The BEs have three nodes and for each node there are three components of displacement u_j^k and traction p_j^k. The shape functions N^k used for these approximations are

$$N^1=\xi_1,\quad N^2=\xi_2,\quad N^3=1-\xi_1-\xi_2 \tag{3}$$

The same shape functions are used to approximate the boundary geometry

$$x_j=\sum_{k=1}^{3}N^k x_j^k \tag{4}$$

where x_j^k are the node coordinates. The same functions are also used to interpolate displacements and tractions for the IBEs

$$u_j=\sum_{k=1}^{Np}N^k u_j^k,\quad p_j=\sum_{k=1}^{Np}N^k p_j^k \tag{5}$$

Each IBE has Np nodes and not the three that the BEs have. The IBE geometry, on the other hand, is approximated by special mapping functions, as discussed in more detail in Section 3.

By substituting Eqs. (2) and (5) in Eq. (1), expression (6) is obtained

$$\begin{aligned}c_{ij}(y)u_j(y)+\sum_{e=1}^{N_{BE}}\left\{\sum_{k=1}^{3}[\Delta p_{ij}^{ek}u_j^k]\right\}+\sum_{e=1}^{N_{IBE}}\left\{\sum_{k=1}^{Np}[\Delta^{\infty}p_{ij}^{ek}u_j^k]\right\}\\=\sum_{e=1}^{N_{BE}}\left\{\sum_{k=1}^{3}[\Delta u_{ij}^{ek}p_j^k]\right\}+\sum_{e=1}^{N_{IBE}}\left\{\sum_{k=1}^{Np}[\Delta^{\infty}u_{ij}^{ek}p_j^k]\right\}\end{aligned} \tag{6}$$

N_{BE} is the number of BEs and N_{IBE} is the number of IBEs. For BEs

$$\Delta p_{ij}^{ek}=\int_{\gamma_e}|J|N^k p^*_{ij}(x,y)\,d\gamma_e,\quad \Delta u_{ij}^{ek}=\int_{\gamma_e}|J|N^k u^*_{ij}(x,y)\,d\gamma_e \tag{7}$$

In Eq. (7), γ_e represents the domain of element e in the local coordinate system and the global system of coordinates is transformed to the local one by the Jacobian $|J|=2A$, where A is the element area in the global system. On the other hand, for IBEs

$$\Delta^{\infty}p_{ij}^{ek}=\int_{\gamma_e}|^{\infty}J|N^k p^*_{ij}(x,y)\,d\gamma_e,\quad \Delta^{\infty}u_{ij}^{ek}=\int_{\gamma_e}|^{\infty}J|N^k u^*_{ij}(x,y)\,d\gamma_e \tag{8}$$

Eq. (8) is analogous to Eq. (7), and the calculation of Jacobian $|^{\infty}J|$ is discussed in Section 3. Integrals of Eqs. (7) and (8) are calculated by standard BEM techniques. Non-singular integrals are evaluated numerically by using integration points. The singular ones, on the other hand, are evaluated by the technique