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Should sophisticated expectations facilitate reaching equilibrium behavior?

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ABSTRACT

This paper develops a Cournot oligopoly best-reply dynamic with multi-order extrapolative expectations and examines the effect of these sophisticated expectations on the long-term equilibrium behavior. Interestingly enough, it demonstrates that more sophisticated expectations which require more effort for foresight should not necessarily lead to more stable equilibrium behavior than less sophisticated expectations which need less effort.

1. Introduction

Much literature has already emerged concerning convergence property of dynamic Cournot oligopoly models [4,7,13,24,26], which characterizes the long-term behavior of oligopolies. However, an underlying hypothesis of dynamic Cournot models is that each firm forms its expectation with respect to the rivals' newest outputs equal to these in the immediately previous time period. Such simple expectations are sometimes too native and even somewhat unrealistic in that firms may collect available information as possible through all channels to learn much about its rivals and finally to form sophisticated expectations. Fisher [7] emphasizes a similar fact: "All discussions of the Cournot model allow the sellers to receive new information as to the outputs of their rivals. No seller is ever assumed to look once and for all at other outputs and then never to look again. Rather, he is assumed to look, then adjust, then look again, and so forth. Cournot oligopolists may be stupid, but they are not stubborn".

In order to overcome this drawback, some sophisticated expectations that are usually characterized by learning mechanisms have been introduced into dynamic oligopoly models. For example, Huck et al. [15] introduce such inertia into the best reply dynamic for a Cournot duopoly game that each firm adjusts its reaction function with a reluctance probability. Okuguchi [19] analyzes the continuous adjustment process in a Cournot oligopoly game in which the rivals' expectations for each firm form adaptively. Bischi and Kopel [3], Gao et al. [12], Kebriaei et al. [17] and Szidarovszky et al. [23] study some discrete adjustment systems with adaptive expectations under Cournot oligopoly games as well. Kamalinejad et al. [16] discuss adjustment dynamics in a Cournot oligopoly market with the rivals' expectations forming with linear regression and recursive weighted least-squares learning method. Naimzada and Tramontana [18] analyze a consumer dynamic choice model with a simple least squared learning mechanism. Gao et al. [10,9] consider discrete nonlinear oligopoly adjustment dynamics with sequential decisions so that the latter firms are able to observe the former ones' latest outputs at every time period. Dawid and Heitmann [5] analyze a best-reply dynamic for Cournot duopoly in which during every time period each firm forms multi-level naive expectations toward their rivals' outputs.

In line with such studies, this paper introduces another type of sophisticated expectations which is referred to as extrapolative expectations. With extrapolative expectations, firms are able to form more reasonable foresight for their expectations toward their rivals' outputs through the technology of Taylor approximation. Quandt [21] may be the first to introduce this learning mechanism into a continuous Bertrand adjustment dynamic, and recently Shamma and Arslan [22] include this mechanism into two continuous classes of evolutionary game dynamics, the fictitious dynamic and the gradient dynamic. The analysis of discrete economic systems is rather different from that of continuous ones and is of great interest and difficulty. To the best of our knowledge, the examination of the relation between extrapolative expectations and convergence under discrete Cournot oligopoly

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dynamics and even general discrete economic systems has been absent, except [8,20]. The crucial difference between the current study and [8] is that this paper employs a simple method relating with the use of naive expectations to characterize extrapolative foresight, while [8] forms extrapolative expectations using the information about rivals' newest outputs. Also, our study is remarkably different from [20], which describes extrapolative expectations straight through historical outputs.

The contributions of our paper can be mainly summed as follows. For one thing, we employ a method to deal with the issue in multi-order extrapolative expectations and develop a related new best-reply dynamic. For another, through comprehensively examining the global convergence of this best-reply dynamic, we indicate unexpectedly that dependent on the order number of extrapolative expectations, more sophisticated expectations may facilitate or hinder the global convergence. In other words, carrying out more sophisticated extrapolative expectations with more efforts should not necessarily bring more desirable outcomes of stabilizing equilibrium behavior.

This paper proceeds as follows. Section 2 gives the basic best-reply dynamic. Section 3 introduces one-order and two-order extrapolative expectations and develops the related best-reply dynamics with extrapolative foresight. Section 4 analyzes and compares global convergence of such best-reply dynamics. Section 5 generalizes the best-reply dynamic and related results into the case of multi-order extrapolative expectations. Section 6 extends our main results into the Cournot oligopoly with general inverse demand and cost functions. Section 7 concludes this paper.

2. The basic model

Consider a dynamic Cournot oligopoly market in which n firms simultaneously update their outputs of homogeneous goods at each discrete time period. Denote by $q_i(t+1)$ firm i's output to be determined at time period $t+1$ ($i = 1, 2, \dots, n$). At each latest period, because of limited rationality and partial knowledge about rivals' outputs, any firm will have to form expectations concerning these outputs in determining its profit-maximizing output. Firm i takes the following adjustment process which is usually referred as the best-reply dynamic

$$q_i(t+1) = \arg\max_{q_i} \pi_i(q_1^e(t+1), \dots, q_{i-1}^e(t+1), q_i, q_{i+1}^e(t+1), \dots, q_n^e(t+1)) \quad (1)$$

where $q_j^e(t+1)(j \neq i)$ denotes firm i's expectations with respect to firm j's output at time period $t+1$.

Adopting the following simple linear inverse demand function and quadratic cost function for firm i at time period $t+1$,

$$p(t+1) = a - b\left[q_i + \sum_{j\neq i}^{n} q_j^e(t+1)\right], \quad a, b > 0 \quad (2)$$

and

$$C_i(q_i) = (d_i/2)q_i^2 + c_iq_i + g_i, \quad d_i, g_i, c_i > 0 \quad (3)$$

one can obtain firm i's expected profit

$$\pi_i(q_1^e(t+1), \dots, q_i, \dots, q_n^e(t+1)) = p(t+1)q_i - (d_i/2)q_i^2 - c_iq_i - g_i. \quad (4)$$

Solving $\frac{\partial \pi_i(q_1^e(t+1),\dots,q_i,\dots,q_n^e(t+1))}{\partial q_i} = 0$ yields the best-reply dynamic for firm i

$$q_i(t+1) = \frac{a-c_i}{2b+d_i} - \frac{b}{2b+d_i}\sum_{j=1,j\neq i}^{n} q_j^e(t+1) \quad (5)$$

that is,

$$q(t+1) = (I-A)q^e(t+1) + v \quad (6)$$

in which $q(t) = (q_1(t), \dots, q_n(t))^T$, $v = \left(\frac{a-c_1}{2b+d_1}, \dots, \frac{a-c_n}{2b+d_n}\right)^T$, I is an identity matrix, and $A = (a_{ij})_{n\times n}$ with $a_{ij} = \frac{b}{2b+d_i}$ and $a_{ii} = 1, i \neq j, \ i, j = 1, \dots, n$.

With naive expectations, firm i forms expectations toward any competitor's latest output simply equal to the adjusted one in the immediately preceding period [3,8,16], i.e., $q_j^e(t+1) = q_j(t)$. The best-reply dynamic with naive expectations thus takes the form of

$$q_i(t+1) = \frac{a-c_i}{2b+d_i} - \frac{b}{2b+d_i}\sum_{j=1,j\neq i}^{n} q_j(t) \quad (7)$$

namely

$$q(t+1) = (I-A)q(t) + v. \quad (8)$$

3. One-order and two-order extrapolative expectations

Even though traditional naive expectations toward competitors' outputs simplify firms' expectations in adjustment processes, it is myopic and even inconsistent with reality in that rivals' outputs will not remain the same as in the immediately preceding period unless the stable equilibrium behavior is approached. To make up this type of unrealistic expectations, we assume all firms share the homogeneous extrapolative expectations with respect to competitors' outputs in the following way of one-order Taylor approximation [1,21]

$$q_j^e(t+1) = q_j(t+\lambda) \approx q_j(t) + \lambda\dot{q}_j(t) \quad (9)$$

where $0 \leq \lambda < 1$ is sufficiently small and measures the level of extrapolative expectations. The parameter λ also measures a firm's foresight ability, and in particular the limiting cases $\lambda = 0$ and $\lambda \to 1$ denote naive expectations and perfect expectations, respectively. Hence, we use the terms "extrapolative foresight" and "extrapolative expectations" interchangeably. Obviously, more realistic outputs expectations will be approached as the parameter λ becomes larger.

Note that one-order extrapolative expectations use rivals' information about their outputs in the near future to characterize extrapolative foresight, which requires that each firm is somewhat more rational. Similar with [7], the continuous equivalent of the best-reply dynamic (7) with naive expectations takes the form of

$$\dot{q}_j(t) = \frac{a-c_j}{2b+d_j} - \frac{b}{2b+d_j}\sum_{k=1,k\neq j}^{n} q_k(t) - q_j(t). \quad (10)$$

One-order extrapolative expectations can be thus described by

$$q_j^e(t+1) = q_j(t+\lambda) \approx \frac{\lambda(a-c_j)}{2b+d_j} - \frac{\lambda b}{2b+d_j}\sum_{k=1,k\neq j}^{n} q_k(t) + (1-\lambda)q_j(t) \quad (11)$$

namely, $q^e(t+1) \approx (I-\lambda A)q(t) + \lambda v$. The best-reply dynamic (6) becomes

$$q(t+1) = (I-A)q^e(t+1) + v = (I-A)(I-\lambda A)q(t) + \lambda(I-A)v + v. \quad (12)$$

Note that during every time period, each firm first forms naive expectations and then uses the related derivative derived from naive expectations to implement the extrapolative foresight in Eq. (9).