



New structural properties of supply chains with price-only contracts



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ABSTRACT

We show that the manufacturer's problem in two-stage push supply chains with price-only contracts can be reformulated as a newsvendor problem with a modified demand distribution. This reformulation provides additional managerial insights, facilitates the solution of a problem with competing retailers under more general assumptions and reduces a number of three-stage problems to equivalent two-stage problems.

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1. Introduction

We consider two-stage push supply chains with a single retailer, a single manufacturer and a price-only contract. In the basic fixed retail price model, studied by Lariviere and Porteus [4], to be called LP in the sequel, the manufacturer acts as a Stackelberg leader and determines the wholesale price. Subsequently, the retailer, who faces an uncertain demand, determines the order quantity in the newsvendor framework. Afterwards, the random demand is realized. We also analyze an extension of this model with multiple competing retailers studied in [7] and show that the problem is solvable under more general assumptions on the random demand distribution.

For the model with one retailer, the retail price is $p > 0$, the manufacturer's unit cost is $c > 0$, $c < p$, the wholesale price is w , $c \leq w \leq p$, and the retailer order quantity is q . We assume zero salvage price and zero shortage cost. All supply chain participants are assumed to be risk-neutral. For a nondecreasing function $s(x)$, we define the elasticity of $s(x)$ with respect to x as $Es(x) = \frac{xs'(x)}{s(x)}$ and the elasticity of the slope of $s(x)$ with respect to x as $Es''(x) = \frac{xs''(x)}{s'(x)}$.

The random demand for the end product, x , has cdf $F(x)$, $\bar{F}(x) = 1 - F(x)$, and pdf $f(x)$ with support on $[0, \infty)$ so that $f(x) = 0$ for $x < 0$ and $f(x) > 0$ for $x \geq 0$. We assume that $f(x)$ is twice differentiable for $x > 0$ and that $\lim_{x \rightarrow \infty} f(x)x = 0$. The failure rate is defined as $h(x) = f(x)/\bar{F}(x)$ and the generalized failure rate as $g(x) = xh(x)$. F is IFR (increasing failure rate) if

$h'(x) \geq 0$ and F is DFR (decreasing failure rate) if $h'(x) \leq 0$ [1]; F is IGFR (increasing generalized failure rate) if $g'(x) \geq 0$ (LP) and, similarly, F is DGFR (decreasing generalized failure rate) if $g'(x) \leq 0$. The partial expected value of x with upper limit z is defined as $G(z) = \int_0^z xf(x)dx$.

In the next section, we study a supply chain with a single retailer and a single manufacturer and its extension to a supply chain with competing retailers.

2. A two-stage supply chain with a fixed retail price and one retailer

We assume in this section that the pdf $f(x)$ has support on $[A, \infty)$, $A \geq 0$. For a fixed w , the retailer's newsvendor profit maximization problem is

$$\max_{q \geq A} \Pi_r(q) = pS(q) - wq, \quad (1)$$

where $S(q) = E[\min(q, x)]$ denotes the expected sales given q ; the maximization in (1) is restricted to $q \geq A$ because the random demand x is at least A . The unique optimal order quantity is given by $q(w) = \bar{F}^{-1}\left(\frac{w}{p}\right)$ (observe that since $w \leq p$, $q(w) = \bar{F}^{-1}\left(\frac{w}{p}\right) \geq \bar{F}^{-1}(1) = A$). Since $w = p\bar{F}(q(w))$, the optimal retailer profit at $q(w)$ is $\Pi_r(q(w)) = pG(q(w))$. Given $q(w)$, the manufacturer's profit maximization problem is $\max_w \Pi_m(w) = (w - c)q(w)$ and it can be reformulated in terms of q as

$$\max_{q \geq A} \Pi_m(q) = [p\bar{F}(q) - c]q. \quad (2)$$

LP proved that if F is IGFR, then $\Pi_m(q)$ is unimodal and the problem $\max_{q \geq A} \Pi_m(q)$ has a unique optimal solution q^* .

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In contrast to LP, we do not require that F is IGFR. In order to simplify the exposition somewhat, we assume that $g(x) \leq 1$ for all $x \geq A$. We show next that the unique optimal solution q^* of the manufacturer's problem (2) is also the solution of a modified single-stage newsvendor problem stated as

$$\max_{q \geq A} \Pi_c(q) = pS_M(q) - cq, \quad (3)$$

where $S_M(q) = E_{F_M}[\min(q, x)] = \int_0^q \bar{F}_M(x)dx$. F_M is a modified distribution defined as

$$F_M(x) = \begin{cases} 0, & \text{if } x \leq 0, \\ g(A^+) \frac{x}{A}, & \text{if } x \in (0, A], \\ 1 - \bar{F}(x)[1 - g(x)], & \text{if } x > A, \end{cases} \quad (4)$$

where $g(A^+) = \lim_{x \rightarrow A^+} g(x)$ (the modified distribution F_M was originally proposed in [3]).

Our main result in this section is that problem (2) has a unique optimal solution (proved by LP for IGFR distributions) (and also that problem (3) has a unique solution) under the following less restrictive assumption:

$$(A1) \quad Ef(x) + 2 = \frac{xf'(x)}{f(x)} + 2 \geq 0 \text{ for } x > A.$$

The assumption (A1) was stated as condition (C2) in [9] and as condition (D4)(i) (with a strict inequality) in [8]. Ziya et al. [9] showed that (A1) implies the concavity of the manufacturer's profit function $\Pi_m(q)$. This assumption will also be used to facilitate the solution of a two-stage supply chain with competing retailers later in this section.

Theorem 1 (Solvability of the Manufacturer's Problem). *If (A1) holds and $g(x) \leq 1$ for all $x \geq A$, then the modified distribution F_M given by (4) is well defined. Moreover, the profit functions in (2) and (3) are unimodal on $[A, \infty)$ and the unique optimal solutions q^* of problems (2) and (3) exist and are identical. If $g(A^+) \geq 1 - \frac{c}{p}$, then $q^* = A$ and if $g(A^+) < 1 - \frac{c}{p}$, then $q^* > A$, $g(q^*) < 1$ and q^* satisfies the condition*

$$p\bar{F}(q^*)[1 - g(q^*)] = c. \quad (5)$$

Proof of Theorem 1. $F_M(x)$ is continuous on $(-\infty, \infty)$ because $F_M(A^-) = F_M(A) = F_M(A^+) = g(A^+)$, $\lim_{x \rightarrow \infty} \bar{F}(x) = 0$ and $\lim_{x \rightarrow \infty} \bar{F}(x)g(x) = \lim_{x \rightarrow \infty} f(x)x = 0$. $F_M(x)$ is nondecreasing on $(-\infty, \infty)$ if and only if $F(x)[1 - g(x)] = \bar{F}(x) - f(x)x$ is nonincreasing for $x > A$. This is in turn equivalent to (A1). Thus, since $F_M(x)$ is continuous and nondecreasing on $(-\infty, \infty)$, the modified distribution F_M is well defined.

The pdf for the modified distribution F_M is given as

$$f_M(x) = \begin{cases} 0, & \text{if } x \leq 0, \\ g(A^+) \frac{1}{A}, & \text{if } x \in (0, A), \\ \frac{\bar{F}(x)}{x} \{g'(x)x + g(x)[1 - g(x)]\}, & \text{if } x > A. \end{cases}$$

Since $\int_l^u \bar{F}(x)[1 - g(x)]dx = u\bar{F}(u) - l\bar{F}(l)$, for $q > A$ we have that

$$\int_A^q \bar{F}_M(x)dx = \int_A^q \bar{F}(x)[1 - g(x)]dx = q\bar{F}(q) - A\bar{F}(A).$$

Thus,

$$S_M(q) = \int_0^q \bar{F}_M(x)dx = \begin{cases} q - g(A^+) \frac{q^2}{2A}, & \text{if } q \in (0, A], \\ q\bar{F}(q) - g(A^+) \frac{A}{2}, & \text{if } q > A. \end{cases}$$

Since (A1) implies that $\bar{F}(x)[1 - g(x)]$ is nonincreasing for $x > A$, $d\Pi_m(q)/dq = d\Pi_c(q)/dq = p\bar{F}(q)[1 - g(q)] - c$ is nonincreasing

for $q > A$. Thus, $d^2\Pi_m(q)/dq^2 = d^2\Pi_c(q)/dq^2 \leq 0$ for $q > A$. Consequently, $\Pi_m(q)$ and $\Pi_c(q)$ are concave on (A, ∞) and thus unimodal on $[A, \infty)$.

Therefore, if $g(A^+) \geq 1 - \frac{c}{p}$, then $d\Pi_m(A^+)/dq = d\Pi_c(A^+)/dq = p\bar{F}(A)[1 - g(A^+)] - c = p[1 - g(A^+)] - c \leq 0$. Thus, since $d\Pi_m(q)/dq$ and $d\Pi_c(q)/dq$ are nonincreasing for $q > A$, $d\Pi_m(q)/dq \leq 0$ and $d\Pi_c(q)/dq \leq 0$ for $q > A$ which implies that the unique optimal solution of both problems is given by $q^* = A$. On the other hand, if $g(A^+) < 1 - \frac{c}{p} < 1$, then, since $d\Pi_m(A^+)/dq = d\Pi_c(A^+)/dq = p[1 - g(A^+)] - c > 0$ and $\lim_{x \rightarrow \infty} d\Pi_m(x)/dq = \lim_{x \rightarrow \infty} d\Pi_c(x)/dq = -c < 0$, the unique optimal solution q^* is such that $q^* > A$, $g(q^*) < 1$ and it satisfies the first order optimality condition $d\Pi_m(q^*)/dq = d\Pi_c(q^*)/dq = 0$ which can be written as (5). This completes the proof.

If F is IGFR then $g'(x) \geq 0$ for $x > A$ or, equivalently, that $\frac{xf'(x)}{f(x)} + g(x) + 1 \geq 0$ for $x > A$. Thus, since we assumed that $g(x) \leq 1$ for $x \geq A$, (A1) holds so that the IGFR assumption on F implies (A1). Xu and Bisi [8] state that a non-IGFR distribution given in Example 2 in Ziya et al. [9] satisfies (A1) which shows that (A1) is satisfied by some non-IGFR distributions. Paul [6] and Xu and Bisi [8] obtained additional conditions, satisfied by some non-IGFR distributions, under which $\Pi_m(q)$ is unimodal.

The following example presents a DGFR distribution that satisfies (A1) and for which problem (3) is solvable.

Example 1. Let $f(x) = \frac{a}{x(a+\ln x)^2} > 0$, $a \geq 2$ with support $[1, \infty)$. Thus, $g(x) = \frac{1}{a+\ln x}$ is decreasing for $x \in [1, \infty)$. Since $a \geq 2$, (A1) holds for $x \in [1, \infty)$.

We can use the modified distribution F_M to relate condition (5) to an analogous condition for the single-stage newsvendor problem $\max_q \Pi_0(q) = pS(q) - cq$. The well-known fractile solution of this problem can be written as $\bar{F}(q^*) = \frac{c}{p}$. Since $S(q)$ represents the expected sales and $\bar{F}(q) = S'(q)$ represents the marginal expected sales, the fractile formula states that the marginal expected sales are equal to the profit margin ratio $\frac{c}{p}$. Similarly, since $\bar{F}_M(x) = \bar{F}(x)[1 - g(x)]$ for $x > A$, condition (5) can be written as $\bar{F}_M(q^*) = \frac{c}{p}$. Therefore $S_M(q)$ represents the expected sales for the problem (3), $\bar{F}_M(q) = S'_M(q)$ represents the corresponding marginal expected sales and condition (5) states that the marginal expected sales for problem (3) are equal to the profit margin ratio $\frac{c}{p}$.

The following result extends Theorem 4 in LP (who assumed that F is IGFR which implies (A1)) on the division of supply chain profits by specifying when the manufacturer captures less than half of the total supply chain profit.

Proposition 1 (Division of Supply Chain Profits). *If (A1) holds and $g(A^+) < 1 - \frac{c}{p}$, then the profit ratio is $\frac{\Pi_m^*}{\Pi_r^*} = EG(q^*)$. Thus, if F is IFR or $G(x)$ is convex for $x \geq 0$ such that $g(x) \leq 1$, then $\Pi_m^* \geq \Pi_r^*$; if $G(x)$ is concave for $x \geq 0$ such that $g(x) \leq 1$, then $\Pi_m^* \leq \Pi_r^*$.*

Proof of Proposition 1. It follows from Theorem 1 that, if $g(A^+) < 1 - \frac{c}{p}$, then the equilibrium order quantity q^* is such that $q^* > A \geq 0$, $g(q^*) < 1$ and it satisfies the first order condition (5). Thus, $\Pi_r^* = pG(q^*)$, $\Pi_m^* = [p\bar{F}(q^*) - c]q^* = p\bar{F}(q^*)g(q^*)q^*$ and

$$\frac{\Pi_m^*}{\Pi_r^*} = \frac{p\bar{F}(q^*)g(q^*)q^*}{pG(q^*)} = \frac{[f(q^*)q^*]q^*}{G(q^*)} = \frac{G'(q^*)q^*}{G(q^*)} = EG(q^*).$$

If $G(x)$ is convex (concave) for $x \geq 0$ such that $g(x) \leq 1$, then $G(0) - G(x) \geq (\leq) G'(x)(0 - x)$, which can be written as $G'(x)x \geq (\leq) G(x)$, or, equivalently as $EG(x) \geq (\leq) 1$, and thus, since $q^* > 0$ and $g(q^*) < 1$, $\frac{\Pi_m^*}{\Pi_r^*} \geq (\leq) 1$. This completes the proof.

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