Operations Research Letters 45 (2017) 170–174

Contents lists available at ScienceDirect

Operations Research Letters

journal homepage: www.elsevier.com/locate/orl

Capacitated assortment and price optimization for customers with disjoint consideration sets

Rui Chen, Hai Jiang *

Department of Industrial Engineering, Tsinghua University, Beijing 100084, China

ARTICLE INFO

Article history:
Received 12 September 2016
Received in revised form
21 January 2017
Accepted 21 January 2017
Available online 4 February 2017

Keywords:
Assortment optimization
Price optimization
Multinomial logit model
Disjoint consideration set

ABSTRACT

We study the capacitated assortment and price optimization problem for customers with disjoint consideration sets. The objective is to find the revenue maximizing set of products and their prices subject to a capacity constraint on the total display space of the offered products. We formulate the problem as a mathematical program and demonstrate its NP-hardness. We propose a fully polynomial-time approximation solution scheme and show that when the weights of the products are identical, our approach yields the optimal solution.

1. Introduction and related literature

It has been recognized that in many business situations, customers can be classified into several distinct segments, each of which can be identified by a unique consideration set that does not overlap with one another; see, e.g., [13,19,17]. For example, customers to a car dealership can be classified into segments based on the types of products, or types of vehicles, they want to buy: a segment that is interested in sedans and another segment that is interested in Sport Utility Vehicles (SUVs). Since the segment that is interested in sedans is unlikely to buy SUVs, while the segment that is interested in SUVs is unlikely to buy sedans, the consideration sets of these two segments do not overlap with each other. Because the different types of vehicles share the same display space within the dealership, the objective of the dealership is to find the revenue maximizing set of vehicles as well as their prices to offer subject to a capacity constraint on the total display space of the offered products. This problem is called the Capacitated Assortment and Price Optimization Problem (CAPOP) for customers with disjoint consideration sets.

The CAPOP for customers with disjoint consideration sets is a problem that has not been thoroughly studied in the literature. For the special case when there is only a single customer segment and the weights of the products are identical, Chen and Hausman [6] model customer choice using the multinomial logit (MNL) model and formulate the CAPOP as a non-linear program with discretized prices, whose coefficient matrix is totally unimodular. Later, Wang [18] develops an efficient algorithm to obtain the optimal assortment and price when the prices of the products can be any arbitrary real number. Recently, Besbes and Sauré [3] study the competition among retailers who compete in both assortment and price, and show that there exists one equilibrium for competitive retailers who offer non-overlapping products. When there is no capacity constraint, Gallego and Topaloglu [7] propose a linear programming based method to obtain the optimal solution to the joint assortment and price optimization problem under the nested logit model. Moreover, Alptekinoğlu and Semple [1] and Jagabathula and Rusmevichientong [11] investigate the assortment and price optimization problem under the exponomial choice and non-parametric models, respectively.

In this paper, we investigate the CAPOP for customers with disjoint consideration sets. Specifically, we model customers' purchase behavior using the *multinomial logit model with disjoint consideration sets*, or the MNLD model. The MNLD model is first introduced by Liu and Van Ryzin [14] and has been widely used in the literature of network revenue management; see, e.g., [19,14,20]. It first partitions products into several non-overlapping groups, each of which corresponds to the disjoint consideration set for a customer segment. Customers in a particular segment only consider purchasing products in one of the consideration sets, following the standard MNL model. In the car store example, there are two customer segments whose consideration sets are composed of sedans and SUVs with different brands and configurations, respectively. Note that when there is only one customer segment with a

* Corresponding author.
E-mail address: haijiang@tsinghua.edu.cn (H. Jiang).

http://dx.doi.org/10.1016/j.orl.2017.01.009

consideration set of all the products, the MNLD model reduces to the standard MNL model.

We summarize the contributions of this research as follows: (a) To the best of the authors' knowledge, this research is the first of its kind to investigate the CAPOP for customers with disjoint consideration sets; (b) we model customers' purchase behavior using the MNLD model and demonstrate the NP-hardness of the corresponding revenue maximization problem; (c) We develop a fully polynomial-time approximation solution scheme based on dynamic programming, where a series of multiple-choice binary fractional knapsack problems are solved; and (d) We show that when the weights of the products are identical, our approach finds the optimal solution.

2. Modeling framework

Assume that the retailer, or the dealership in our previous example, has a total of n possible products, indexed by $N = \{1, 2, \dots, n\}$, and there are L customer segments with disjoint consideration sets. Under the MNLD model, these products are first partitioned into L disjoint groups, denoted as $N_1, N_2, \dots, N_L$ where $\cup_{l=1}^{L} N_l \subseteq N$ and $N_l \cap N_{l'} = \emptyset$ for all $l \neq l'$. Customers in segment $l \in \{1, 2, \dots, L\}$ come to the system with probability α^l and only consider purchasing products in set N_l or leave immediately. This implies that $\sum_{l=1}^{L} \alpha^l = 1$. We further assume that for each product, there is a set of pre-determined price levels, which is not unreasonable because most, if not all, prices of cars end in 9. For example, the price of a 2015 Volkswagen Jetta 1.8T SE may be 21,999 dollars. We assume that each product can be offered at m price levels, indexed by $M = \{1, 2, \dots, m\}$. Our notation implies that the number of possible price levels for each product is the same, which is only for notational brevity and our results can be easily applied to the case when the numbers of price levels differ across products. For each product $i \in N$, let $w_i \in \mathbb{Z}^+$ be its weight, or required display space, and p_{ij} be its price at the jth price level. Given a capacity constraint $c \in \mathbb{Z}^+$ on the total weight of the products, the retailer wishes to find the subset of products as well as their corresponding price levels that maximizes the expected revenue.

Under such a setting, let $S_{ij} = 1$ indicate that product i at price level j is offered; and $S_{ij} = 0$, otherwise. This means that $\sum_{j \in M} S_{ij} \leq 1$. Thus, the CAPOP can be treated as a modified variant of the classical assortment problem with side constraints. Additionally, for product $i \in N$ with price p_{ij}, its utility is denoted as v_{ij}, where $v_{ij} \leq v_{ij'}$ holds if $p_{ij} \geq p_{ij'}, \forall i \in N$ to reflect the fact that when the price of the product increases, it becomes less attractive to customers. Moreover, as a convention, we set the utility for the no-purchase option as one.

Given the above, for a feasible assortment $\boldsymbol{S} = [S_{ij}]^{n \times m}$, a customer in segment l will purchase product i at price level j with a probability of

$$P_{ij}(\boldsymbol{S}) = \begin{cases} 0, & i \notin N_l; \\ v_{ij}S_{ij} \Big/ \left(1 + \sum_{i \in N_l} \sum_{j \in M} v_{ij} S_{ij}\right), & i \in N_l; \end{cases} \tag{1}$$

and he or she will leave without purchasing anything with probability $P_0(\boldsymbol{S}) = 1/(1 + \sum_{i \in N_l} \sum_{j \in M} v_{ij} S_{ij})$. Then, the CAPOP under the MNLD model can be formulated as follows:

$$z^* = \max \quad R(\boldsymbol{S}) = \sum_{l=1}^{L} \alpha^l \left\{ \frac{\sum_{i \in N_l} \sum_{j \in M} p_{ij} v_{ij} S_{ij}}{1 + \sum_{i \in N_l} \sum_{j \in M} v_{ij} S_{ij}} \right\} \tag{2}$$

$$\text{s.t.} \quad \sum_{i \in N} \sum_{j \in M} w_i S_{ij} \leq c, \tag{3}$$

$$\sum_{j \in M} S_{ij} \leq 1, \quad \forall i \in N, \tag{4}$$

$$S_{ij} \in \{0, 1\}, \quad \forall i \in N, j \in M. \tag{5}$$

This problem is a special case of the constrained hyperbolic binary programming problem, whose applications and computational complexity issues are discussed in [15,5]. Note that when $N_l = \{l\}$ for all $l \in \{1, 2, \dots, L\}$, we can have $R(\boldsymbol{S}) = \sum_{l=1}^{L} \sum_{j \in M} (\alpha^l p_{lj} v_{lj} / (1 + v_{lj})) S_{lj}$ because of Eq. (4). In this case, the CAPOP under the MNLD model reduces to a multiple-choice knapsack problem, which is a well-known NP-hard problem; see, e.g., [8,12]. Therefore, the following complexity result is immediately established.

Lemma 1. *The CAPOP under the MNLD model is NP-hard.*

3. Solution approach

For the CAPOP under the MNLD model, it may appear that whether to offer product $i \in N_{l'}$ does not directly affect the revenue obtained from customers in segment $l \neq l'$, because there are no explicit l'-segment related terms in the revenue associated with segment l, that is, $R_l(\boldsymbol{S}_l) = \sum_{i \in N_l} \sum_{j \in M} p_{ij} v_{ij} S_{ij} / (1 + \sum_{i \in N_l} \sum_{j \in M} v_{ij} S_{ij})$ and $\boldsymbol{S}_l = [S_{ij}]^{n_l \times m}$, where $n_l = |N_l|$. However, products in N_l and $N_{l'}$ do interact because they compete for the limited display space and the retailer has to decide the capacity allocated to products in N_l and $N_{l'}$. This prompts us to design a dynamic programming based solution approach to optimally allocate the limited capacity among these products.

To construct the dynamic program, define $\Theta_l(b)$ as the maximum expected revenue that we can obtain if the capacity consumption of offered products in $N_l \cup \cdots \cup N_L$ for customers in segments l to L is bounded by $b \in \mathbb{Z}^+$. Define $R_l(\boldsymbol{S}_l, b_l) = \left\{ \sum_{i \in N_l} \sum_{j \in M} p_{ij} v_{ij} S_{ij} / (1 + \sum_{i \in N_l} \sum_{j \in M} v_{ij} S_{ij}) : \sum_{i \in N_l} \sum_{j \in M} w_i S_{ij} \leq b_l; \sum_{j \in M} S_{ij} \leq 1, \forall i \in N_l \right\}$, then the Bellman equation can be expressed as follows:

$$\Theta_l(b) = \max_{b_l \leq b} \left\{ \alpha^l \max_{\boldsymbol{S}_l \in \Omega_l(b_l)} \{R_l(\boldsymbol{S}_l, b_l)\} + \Theta_{l+1}(b - b_l) \right\},$$
$$\forall l \in \{1, 2, \dots, L\}; b_l, b \in \{0, 1, \dots, c\}, \tag{6}$$

where $\Omega_l(b_l)$ is the set of all feasible assortments that $\Omega_l(b_l) = \left\{ \boldsymbol{S}_l = [S_{ij}]^{n_l \times m} : \sum_{i \in N_l} \sum_{j \in M} w_i S_{ij} \leq b_l; \sum_{j \in M} S_{ij} \leq 1, \forall i \in N_l \right\}$. And the boundary conditions are $\Theta_{L+1}(\cdot) = 0$. We compute the values of $\left\{ \Theta_l(b) : l \in \{1, 2, \dots, L\}; b \in \{0, 1, \dots, c\} \right\}$ and the value of $\Theta_1(c)$ corresponds to maximum expected revenue, z^* in Eq. (2).

In the dynamic program above, we have to enumerate not only the values of $b \in \{0, 1, \dots, c\}$ in each state, but also the feasible assortments formed by the products. Define $R_l(b_l) = \max_{\boldsymbol{S}_l \in \Omega_l(b_l)} \{R_l(\boldsymbol{S}_l, b_l)\}$, which corresponds to the maximum expected revenue we can obtain if the next customer is interested in class l and b_l units of capacity are allocated for products in N_l. We give an alternative characterization of $R_l(b_l)$ in the following lemma, following a transformation technique similar to [7]:

Lemma 2. *Let λ^* be the value of λ that satisfies*

$$\lambda = \max \left\{ \sum_{i \in N_l} \sum_{j \in M} v_{ij}(p_{ij} - \lambda) S_{ij} : \boldsymbol{S}_l \in \Omega_l(b_l) \right\}, \tag{7}$$

then $\lambda^* = R_l(b_l)$. Furthermore, the optimal solution to the problem

$$\max_{\boldsymbol{S}_l \in \Omega_l(b_l)} \left\{ \sum_{i \in N_l} \sum_{j \in M} v_{ij}(p_{ij} - \lambda^*) S_{ij} \right\} \tag{8}$$

corresponds to the optimal solution to $R_l(b_l)$.