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Optimal information disclosure policies in a strategic queueing model

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ABSTRACT

We find the optimal policy for the information disclosure problem of the M/M/1 queue studied by Simhon et al. (2016). Our optimal disclosure policy is as follows: the service provider informs all customers of the queue length when the queue length is above a specified threshold and does not inform them when the queue length is below the threshold.

1. Introduction

The economic analysis of queueing systems with strategic customer behavior has gained a considerable amount of interest since the works of Naor [8] and Edelson and Hildebrand [3] were published. Information on the queue length is an important factor for customers who make the decision whether to join a queue or not. Queueing systems with strategic customer behavior are usually divided into two groups: observable and unobservable queues. In an observable queue customers are informed of the queue length upon arrival, whereas in an unobservable queue customers are not informed of the queue length upon arrival. For more details, refer to Hassin and Haviv [6].

It is important to investigate if it is effective for the service provider (server) to provide the information on the queue length to customers, with the intention to increase the service provider's own profit (revenue). There has been a large amount of research on the effects of the information level on the strategic behavior of customers, and on the service provider's profit. For example, Economou and Kanta [2] considered an M/M/1 queue in which the waiting space of the system is partitioned into compartments with fixed capacity for customers. Before entering, customers are informed which compartment they will enter and/or the position within the compartment. Guo and Zipkin [4] considered an M/M/1 queue with a nonlinear waiting cost function, which is also dependent on the delay sensitivity of the customers. Hassin and Koshman [7] considered an M/M/1 queue where arriving customers cannot observe the queue length, but are only informed if the queue length is either above or below the threshold (i.e., the congestion level is either high or low). Dobson and Pinker [1] developed a stochastic model of a custom-production environment in which customers have different tolerances for waiting. The firm (service provider) has the option to share different amounts of information about the lead time that a customer may incur. For a summary of information control, refer to Section 3.5 of the book by Hassin [5]. In addition, Shone et al. [9] studied the conditions for the equality of effective arrival rates (the rates at which customers join the queue for service) between the observable and unobservable M/M/1 queues.

The objective of this paper is to find the optimal information disclosure policy that the service provider should adopt to maximize its own profit. We assume that the service provider has a fixed income from each customer who joins the queue, so the service provider should maximize the effective arrival rate to maximize its own profit. That is, the objective of the service provider is to maximize the throughput of the system. Simhon et al. [10] studied the optimal information disclosure policies in an M/M/1 queue. They proved that the policy of informing customers about the current queue length when the queue length is below a specified threshold and hiding the information when the queue length is above the threshold, is never optimal.

In this paper, we will find the optimal information disclosure policy in the M/M/1 queue. Our optimal policy is as follows: the service provider informs all customers of the queue length when the queue length is above the specified threshold and does not inform them when the queue length is below the threshold.

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The paper is organized as follows. In Section 2, we describe the model and briefly discuss the customer's equilibrium strategies under a policy adopted by the service provider. In Section 3, we find the optimal information disclosure policy.

2. The model

We consider an $M/M/1$ queue with first-come first-served discipline. Customers arrive according to a Poisson process with rate λ, and they are allowed to decide whether to join or balk upon arrival. The service times are independent and exponentially distributed with mean μ^{-1}. The cost to a customer for staying in the system (either waiting or being served) is C per unit of time. All customers have the same reward R from completion of service, where $R > \frac{C}{\mu}$ (if $R \le \frac{C}{\mu}$, then customers have no incentive to join the queue). We denote by $\rho = \frac{\lambda}{\mu}$ the offered load of the system.

We consider a state-dependent information disclosure policy u, which is represented by a mapping from $\mathbb{Z}^+$ to $[0, 1]$. The value $u(i), i \in \mathbb{Z}^+$, is the probability that the service provider gives the information to an arriving customer when there are i customers in the system (including the customer being served). Let $\mathcal{U}$ be the set of all information disclosure policies. If $u(i) = 1$ for all $i \ge 0$, then this information disclosure policy is denoted by u_+. In this case it is exactly the observable model presented by Naor [8]. If $u(i) = 0$ for all $i \ge 0$, this information disclosure policy is denoted by u_-. In this case it is the unobservable model presented by Edelson and Hildebrand [3].

An arriving customer decides to join or to balk depending on her expected waiting cost and reward. The customers are assumed to know the policy of the service provider when they choose their strategies. A customer will join the queue if the expected waiting cost (including the cost due to her service), given the available information on the queue length, is smaller than the reward, and will balk if it is larger than the reward. We consider a customer's strategy of joining s, which is represented by a mapping from $\mathbb{Z}^+ \cup \{\Delta\}$ to $[0, 1]$. Specifically, $s(i), i \in \mathbb{Z}^+$, is the probability that an arriving customer joins the queue when this customer is informed that there are i customers in the system, and $s(\Delta)$ is the probability that an arriving customer joins the queue when this customer is not informed of the queue length.

Let $N(t)$ be the number of customers in the system at time t. If the policy u is adopted by the service provider and the strategy s is adopted by the customers, then the distribution of the stochastic process $\{N(t) : t \ge 0\}$ is determined. If $N(t)$ has a stationary distribution under (u, s), then the stationary distribution is unique. Let $\pi_{u,s}(i), i \in \mathbb{Z}^+$, denote the stationary distribution of $N(t)$ under (u, s), if it exists. For a given policy u, a strategy s is an *equilibrium strategy* if and only if $N(t)$ has the stationary distribution and the following conditions hold:

- if $i < \frac{R\mu}{C} - 1$, then $s(i) = 1$; (1)
- if $i > \frac{R\mu}{C} - 1$, then $s(i) = 0$; (2)
- if $\dfrac{\sum_{j=1}^{\infty} \pi_{u,s}(j)(1-u(j))j}{\sum_{j=0}^{\infty} \pi_{u,s}(j)(1-u(j))} < \frac{R\mu}{C} - 1$, then $s(\Delta) = 1$; (3)
- if $\dfrac{\sum_{j=1}^{\infty} \pi_{u,s}(j)(1-u(j))j}{\sum_{j=0}^{\infty} \pi_{u,s}(j)(1-u(j))} > \frac{R\mu}{C} - 1$, then $s(\Delta) = 0$. (4)

Eq. (1) means that an informed customer will join the queue if her expected cost for staying in the system is less than the reward R, i.e., $(i+1)\frac{C}{\mu} < R$. Eq. (2) means that an informed customer will balk if her expected cost is greater than R, i.e., $(i+1)\frac{C}{\mu} > R$. If $i = \frac{R\mu}{C} - 1$, then the informed customer is indifferent between joining and balking, and so $s(i)$ can take any value in $[0, 1]$. Eq. (3) means that an uninformed customer will join the queue if her expected cost is less than R. Eq. (4) means that an uninformed customer will balk if her expected cost is greater than R. If $\frac{\sum_{j=1}^{\infty} \pi_{u,s}(j)(1-u(j))j}{\sum_{j=0}^{\infty} \pi_{u,s}(j)(1-u(j))} = \frac{R\mu}{C} - 1$, then uninformed customers are indifferent between joining and balking, and so $s(\Delta)$ can take any value in $[0, 1]$. Note that the conditions of (3) and (4) require $\sum_{j=0}^{\infty} \pi_{u,s}(j)(1-u(j)) \neq 0$. Hence, if $\sum_{j=0}^{\infty} \pi_{u,s}(j)(1-u(j)) = 0$, then $s(\Delta)$ can take any value in $[0, 1]$. In this case, if $\pi_{u,s}(i), i \in \mathbb{Z}^+$ is the steady state distribution of $N(t)$, then the service provider informs all arriving customers of the queue length. Therefore, customers do not need to use the probability $s(\Delta)$.

The service provider's objective is to maximize its own profit generated from service completions. Let $T_{u,s}$ be the long run rate of service completions under (u, s). Then $T_{u,s}$ is given by

$$T_{u,s} = \mu(1 - \pi_{u,s}(0)),$$

if the probability of the server being idle, at the steady state, is $\pi_{u,s}(0)$. For a policy $u \in \mathcal{U}$, let S_u be the set of all equilibrium strategies with respect to u. It can be shown that $S_u \neq \phi$ for every $u \in \mathcal{U}$. The proof can be found in Appendix A. If s is an equilibrium strategy for u, i.e., $s \in S_u$, then $N(t)$ has a unique stationary distribution, $\pi_{u,s}(i), i \in \mathbb{Z}^+$. Hence $\pi_{u,s}(0) > 0$ if s is an equilibrium strategy for u. A policy u^* is *optimal* if and only if

$$T_{u^*,s^*} \ge T_{u,s} \quad \text{for all } u \in \mathcal{U}, s \in S_u, \text{ and } s^* \in S_{u^*}.$$

In the following two examples, we present the equilibrium strategies and the stationary idle probabilities, when the two types of policies, u_+ and u_-, are used by the service provider.

Example 1 (*When the Policy u_+ is Adopted*)**.** Suppose that an arriving customer is informed that there are i customers in the system (at the arrival instant). Consider the two cases where $\frac{R\mu}{C}$ is not an integer and where $\frac{R\mu}{C}$ is an integer, separately. If $\frac{R\mu}{C}$ is not an integer, then $s \in S_{u_+}$ if and only if

$$s(i) = \begin{cases} 1 & \text{if } i \le \left[\frac{R\mu}{C}\right] - 1, \\ 0 & \text{if } i > \left[\frac{R\mu}{C}\right] - 1. \end{cases}$$

Under u_+ and $s \in S_{u_+}$, $N(t)$ is a birth and death process on a finite state space $\{0, 1, \ldots, [\frac{R\mu}{C}]\}$, with birth rate λ from state i to state $i+1$ for all $0 \le i \le [\frac{R\mu}{C}] - 1$ and death rate μ from i to $i-1$ for all $1 \le i \le [\frac{R\mu}{C}]$. This is the standard $M/M/1/[\frac{R\mu}{C}]$ queue. Thus,

$$\pi_{u_+,s}(0) = \frac{1}{\sum_{i=0}^{[\frac{R\mu}{C}]} \rho^i}.$$

If $\frac{R\mu}{C}$ is an integer, then $s \in S_{u_+}$ if and only if $s(i) = 1$ for all $i < \frac{R\mu}{C} - 1$ and $s(i) = 0$ for all $i > \frac{R\mu}{C} - 1$. Under u_+ and $s \in S_{u_+}$, $N(t)$ is a birth and death process on a finite state space $\{0, 1, \ldots, \frac{R\mu}{C}\}$, with birth rate λ from state i to state $i+1$ for $0 \le i \le \frac{R\mu}{C} - 2$ and birth rate $\lambda\, s(\frac{R\mu}{C} - 1)$ for $i = \frac{R\mu}{C} - 1$. The