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Generalized assignment problem: Truthful mechanism design without money

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ABSTRACT

We propose truthful approximation mechanisms for strategic variants of the generalized assignment problem (GAP) in a payment-free environment. In GAP, a set of items has to be optimally assigned to a set of bins without exceeding the capacity of any singular bin. In our strategic variant, bins are held by strategic agents and each agent may hide its willingness to receive some items in order to obtain items of higher values. The model has applications in auctions with budgeted bidders.

1. Introduction

Truthful mechanism design without money under general preferences is a classic topic in social choice theory. Truthfulness ensures that no agent can be better off by manipulating its true preferences. When searching for truthful mechanisms *without money*, one has to look at restricted domains of preferences. The reason for this, is the Gibbard–Satterthwaite theorem which states that any truthful social choice function which selects an outcome among three or more alternatives has to be trivially aligned with the preference of a single agent (namely, the dictator) [7,11]. Thus, exploring domains for which there exist truthful mechanisms is of central importance in the field of social choice theory.

As an example for restricted domains, consider agents with single-peaked preferences. In this domain returning the *median* of the peaks determines a truthful social choice [8]. Another example is the two-sided matching, in which a set of men has a strict preference ordering over a set of women, and vice versa. A matching is an assignment of men to women where each side is assigned to only one element of the other side. The *deferred acceptance algorithm* finds a stable matching which is truthful for the proposing side, but not necessarily truthful for the other side [10].

One way to circumvent the impossibility result is relaxing the social choice function. Procaccia and Tennenholtz introduced the technique of welfare approximation as a means to derive truthful approximation mechanisms without money [9]. This type of approximation is not meant to handle computational intractability, but a method to achieve truthfulness by relaxing the goal of optimizing social welfare (approximating social welfare), and thus circumventing the Gibbard–Satterthwaite impossibility theorem. The approach is to maximize welfare without considering incentives, and refer to this as optimal value. Then it is said that a truthful mechanism returns (at most) an α-approximation of the optimal if its value is always greater than or equal to $1/\alpha$ times the optimal value ($\alpha \geq 1$). Several works, subsequent to the work of Procaccia and Tennenholtz, employ this technique [4,3]. We apply this technique to a novel strategic setting in the following.

1.1. Model

Consider a strategic variant of the generalized assignment problem termed GAP-BS in an environment which is both prior-free and payment-free. In GAP-BS, there are m items J and n bins (knapsacks) I. Each bin i has a capacity C_i and associates a value v_{ij} and a size w_{ij} to any item j. A feasible assignment may allocate a subset of items S to bin i such that $\sum_{j\in S} w_{ij} \leq C_i$. A feasible assignment may assign each item at most once.

In GAP-BS, we assume tuple $T = (\{v_{ij}\}_{ij}, \{w_{ij}\}_{ij}, \{C_i\}_i)$ is public, but each bin is held by a strategic agent. The private information that each agent/bin holds is the set of its compatible items. The compatibility between an agent and an item encodes the

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willingness of the agent to receive the item. In particular, consider a bipartite graph G where one side corresponds to items and the other side corresponds to bins. The edges of $G, E \subseteq I \times J$ represent the compatible item–bin pairs. The private type of a bin i is therefore the set of edges in the graph incident on i, i.e. E_i. A bin i receives value $v_i(S) = \sum_{j\in S:(i,j)\in E_i} v_{ij}$ from package S if $\sum_{j\in S} w_{ij} < C_i$ and 0, otherwise. The total value of a feasible assignment $(S_1, S_2, \ldots, S_n)$ equals the sum of values received by the bins from the assignment: $\sum_{i\in I} v_i(S_i)$. We seek a total value-maximizing algorithm that provides each bin i with incentives to truthfully report its compatible items E_i rather than any $E'_i \subset E_i$. In fact, our results certify that each bin i reports exactly E_i, and has no incentives to report any other set of edges E'_i. However, for the sake of simplicity in the exposition of the results, we focus on untruthful reports that are made by hiding some edges, $E'_i \subset E_i$. In other words, given a truthful mechanism, bins have no incentive to hide their compatibility with some items.

Let $\mathcal{A}$ denote a randomized algorithm which takes instance (T, E) and computes $X \in \{0,1\}^E$, an assignment of items to bins. Algorithm $\mathcal{A}$ is internally randomized; it returns a solution which is randomly chosen according to a lottery over feasible assignments. Thus, the computed assignment may change by running $\mathcal{A}$, twice on the same input. Randomized algorithm $\mathcal{A}$, given any tuple T, should satisfy the following properties.

i. (feasibility) $\forall j \in J, Pr[\sum_{i\in I} X_{ij} \le 1] = 1$ and $\forall i \in I, Pr[\sum_{j\in J} w_{ij}X_{ij} \le C_i] = 1$, where $X \sim \mathcal{A}(T, E)$, for all E.
ii. (incentive compatibility) for all i, E_i, E_{-i}, and any reported $E'_i \subset E_i$, we have $\mathbb{E}[\sum_{j:(i,j)\in E_i} v_{ij}X_{ij}] \ge \mathbb{E}[\sum_{j:(i,j)\in E_i} v_{ij}X'_{ij}]$, where $X \sim \mathcal{A}(T, E)$, and $X' \sim \mathcal{A}(T, E'_i \cup E_{-i})$.

E_{-i} always denotes $E \setminus E_i$. The expectation in *ii* is taken over the coin flips of the algorithm. Note that, the expected value of the bin in both cases is calculated with respect to true item–bin compatibilities, E. We remark that condition *ii* characterizes mechanisms that are dominant strategy incentive compatible. In this paper, for brevity, we refer to these mechanisms as truthful mechanisms or algorithms. To sum, our objective is to propose a randomized algorithm $\mathcal{A}$ for GAP-BS which is truthful, and always returns a feasible assignment whose value approximates the optimal total value as high as possible.

Many real-world decision problems can be modeled by variants of knapsack problems, therefore we believe that our model can be applied broadly. As an example, we refer to the *maximum budgeted allocations* (MBA) problem [2]. In MBA, a set of indivisible items has to be assigned to a set of bidders. Each bidder i reports her willingness to pay b_{ij} for item j by bidding for the item, while she has a budget constraint B_i. Each bidder i on receiving a package S of items, pays $\sum_{j\in S} b_{ij}$. Each bidder i has the rigid constraint B_i on her payment. The goal in MBA is to find a distribution of items among the bidders which maximizes the total revenue (the sum of the payments by the bidders while respecting their budget constraints). MBA arises in auctions with budgeted bidders and has several applications [2].

In MBA, bidders want to get as much as they can without spending more than their budget. For instance, advertisers wish to maximize the impressions, clicks, or sales generated by their advertising, subject to budget constraints. Similarly, bidders who have no direct utility for leftover money (e.g. because the money comes from a corporate budget) will buy as much as possible. This types of bidders are called *value maximizers*, and have recently drawn the attention of researchers in mechanism design [5].

Consider a strategic variant of MBA in which each bidder, in order to obtain a more valuable package of items, strategizes in the following way. Each bidder may strategically hide her interests in buying some items by not bidding for those items. In this setting, the auctioneer wishes to certify that each bidder truthfully reveals her willingness to buy items. In other words, a truthful mechanism in this setting will encourage participation of the bidders in the auction. We model this setting by GAP-BS in which each bidder is represented by a bin, budgets B_i by capacities C_i, the bids b_{ij} by the values of bins for the items v_{ij}, and the payment by a bidder i for item j by the weight of the item on the bin, w_{ij}. Thus, in this setting of GAP-BS, we have $v_{ij} = w_{ij}$ for all i and j. For this problem, since the value density of each item is the same over all bins, we provide a truthful 4-approximation algorithm.

1.2. Discussion about the assumptions

Aside from the applications of the model discussed above, we emphasize that our assumptions (which imply a highly structured domain) are necessary to escape the impossibility results such as the Gibbard–Satterthwaite theorem and its variations. For example, we resort to welfare approximations because as stated by Theorem 1, no deterministic (or randomized) algorithm whose value is optimal, exists for GAP-BS. The lower bounds in Theorem 1 were derived for a different setting with strategic items in the literature, however, we can reproduce and adapt the theorem for our setting.

Theorem 1 (*[4]*). *No truthful deterministic algorithm with an approximation ratio better than* 2 *exists for GAP-BS. Moreover, no truthful-in-expectation randomized algorithm with an approximation ratio better than* 1.09 *exists for GAP-BS.*

Now, we consider a setting in which bins/agents have private values for items. This setting is more general than GAP-BS in that, in this setting, the agents can manipulate their valuations for items. This is in contrast to GAP-BS in which the agents can only hide their valuations for some items by hiding their compatibility with those items. For this general setting, no deterministic (or randomized) truthful algorithm, with an interesting approximation ratio, exists. To see this, consider a simple market with one item, and a set of agents. This market is equivalent to the single-item auction, but without money. We observe that no mechanism without money can find the (true) highest valuation for the item, as the agents can report arbitrarily high values for the item. That is, no truthful algorithm can do anything better than the algorithm which allocates the item to the bin which is uniformly chosen at random. Such an algorithm provides a trivial approximation ratio of $1/n$, n being the number of agents.

In a parallel setting, Dughmi et al. [4] and Chen et al. [3] studied GAP in an environment in which *items* are held by strategic agents. This is in contrast to our assumption that *bins* are held by strategic agents. Hence, the solutions proposed by these authors are not directly applicable to GAP-BS. In GAP each item can be assigned only once, thus the setting studied by Dughmi et al. is appropriate for modeling single-demand bidders who are interested in buying only a single item. However, our model analyzes strategic bins which can model multi-demand bidders, i.e., bidders who are interested in buying multiple items. In particular, the sizes in our model are at the side of strategic agents which properly models the bidders' budgets in MBA problem.

2. Truthful mechanisms for GAP-BS and variants

In addition to GAP-BS, we also analyze two variants, namely the multiple knapsack problem in which each item has the same size and value over bins, and density-invariant GAP in which each item has the same value density (value per size) over the bins. For an extended version of the paper, we refer the reader to Fadaei and Bichler [6].

We observe that the relaxation and rounding technique is applicable to these problems. The relaxation and rounding