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On the impact of suboptimal decisions in the newsvendor model

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ABSTRACT

We study the impact of suboptimal decisions in the newsvendor model, one of the popular inventory models. We establish a lower bound for the deviation of inventory cost from its minimum, when the order quantity is suboptimal. Demonstration of the bound shows the model to be sensitive to suboptimal decisions.

1. Introduction

The newsvendor problem is about stocking decision of a product with uncertain demand, where mismatch between demand and supply attracts penalty. This classic inventory problem was first addressed by Arrow et al. [1]. Owing to its wide applicability, the newsvendor problem attracted attentions of many scholars over the past six decades. Review of their works can be found in Khouja [8], Qin et al. [11], and Choi [3].

Simplest case of the newsvendor problem, known as the classical newsvendor problem, arises when we make certain simplifying assumptions about the demand and supply processes. Key assumptions are: (i) exogenous demand with known distribution, (ii) single procurement of any amount, and (iii) linearity of cost components. See Chapter 10 of Silver et al. [12] for the details. Let X denote the stochastic demand with distribution function F. Let c_o and c_u denote the unit over-stocking and under-stocking costs. Let Q denote the order quantity. Then demand–supply mismatch cost, $C(Q, X)$ and its expected value are given by

$$C(Q, X) = c_o \max\{0, Q - X\} + c_u \max\{0, X - Q\}$$

$$E[C(Q)] = \int_{-\infty}^{Q} c_o(Q - x)\mathrm{d}F(x) + \int_{Q}^{\infty} c_u(x - Q)\mathrm{d}F(x)$$
$$= (c_o + c_u)\left[\xi(\mu - Q) + \int_{-\infty}^{Q} F(x)\mathrm{d}x\right] \quad (1)$$

where $\xi = c_u/(c_o + c_u)$ is referred to as the critical fractile and μ is the mean demand. $E[C(Q)]$ is convex in Q and the optimal solution is given by: $F(Q^*) = \xi$.

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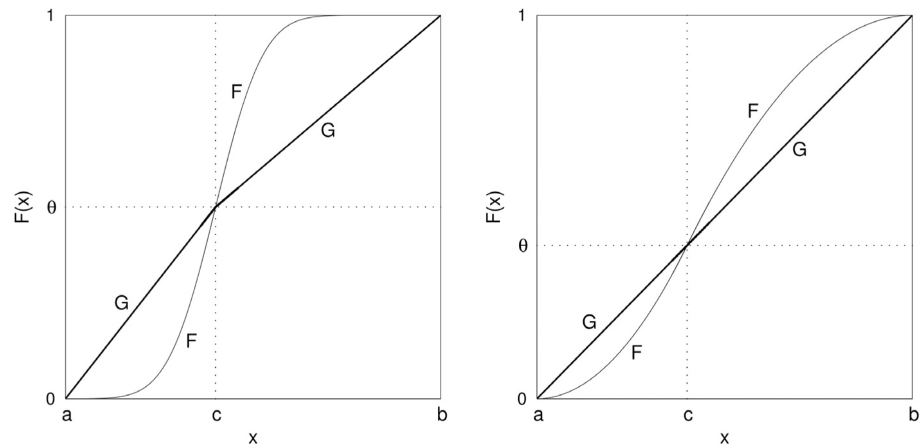
Decision making in the classical newsvendor model requires the knowledge of demand distribution and cost parameters. When estimates of these parameters deviate from their true values, the actual decision (derived using the estimates) deviates from the optimum (calculated using the true values). Due to convex nature of the objective function, any deviation from the optimal decision increases cost. In this context, sensitivity analysis is performed to understand the impacts of (i) suboptimal decisions on expected cost and (ii) parameter estimation error on stocking decision. It shall be noted that the second question is of practical relevance when the impact of suboptimal decisions on expected cost is significant. Some of the popular inventory models, e.g., economic order quantity (EOQ), stochastic (r, Q), and stochastic (s, S) models have been found to be insensitive to suboptimal decisions [10,14,2]. In this sense, we should investigate question-(i) first.

In the newsvendor literature, there are some papers (e.g., [5,4,9,13]) that, to some extent, address question-(ii), i.e., the impact of parameter estimation error on stocking decision. However, question-(i), which is our focus, remains largely unanswered with the exception of one paper. In a recent article, Khanra et al. [7] established a lower bound for cost deviation, i.e., the deviation of expected cost from its minimum value. Demonstration of the lower bound showed the newsvendor model to be sensitive to suboptimal decisions. In a number of scenarios, cost deviation exceeded order quantity deviation, i.e., the deviation of order quantity from its optimal value. This behaviour of the newsvendor model is opposite to that of the EOQ, (r, Q), and (s, S) models.

Khanra et al. [7] assumed the demand to follow symmetric unimodal distribution. In this paper, we study robustness of the newsvendor model to suboptimal decisions when the demand distribution is not necessarily symmetric. In particular, we establish a new lower bound for cost deviation when the demand follows general unimodal distribution. Demonstration of the



(a) $\theta \neq m$.   (b) $\theta = m$.

Fig. 1. $G \in \mathcal{UD}_{a,b,c,\theta}$.

lower bound establishes sensitivity of the newsvendor model to suboptimal decisions.

2. New lower bound for cost deviation

We need to decide measures for cost and order quantity deviations. We choose relative measure, i.e., cost deviation is measured by $\delta_C = (E[C(Q)] - E[C(Q^*)])/E[C(Q^*)]$, and order quantity deviation is measured by $\delta_Q = (Q - Q^*)/Q^*$. These measures are unit-less fractions, hence, easy to compare. Using (1), we can express δ_C as a function of δ_Q as follows.

$$\delta_C(\delta_Q) = \frac{\int_{Q^*}^{Q^*(1+\delta_Q)}\{F(x) - \xi\}\mathrm{d}x}{\xi(\mu - Q^*) + \int_{-\infty}^{Q^*} F(x)\mathrm{d}x}. \quad (2)$$

Let us adjust the unit of cost so that $c_o + c_u = 1$. Then the numerator of $\delta_C(\delta_Q)$ in (2) is the absolute deviation of cost, denoted by $\Delta_C(\delta_Q)$, and the denominator is the minimum mismatch cost, $E[C(Q^*)]$. We establish the lower bound for $\delta_C(\delta_Q)$ by combining a lower bound of $\Delta_C(\delta_Q)$ and an upper bound of $E[C(Q^*)]$. To obtain these bounds for unimodal demand, first we need to characterize such distributions.

2.1. Unimodal demand distributions

We call a distribution F to be unimodal if there exists $c \in \mathbb{R}$ such that F is convex in $(-\infty, c]$ and concave in $[c, \infty)$ [6]. Since we are dealing with demand distributions, we can safely assume a bounded support for the distribution. Let us denote the family of unimodal distributions with support $[a, b]$, mode c, and $F(c) = \theta$ by $\mathcal{UD}_{a,b,c,\theta}$. Let $r = a/b$ denote ratio of the demand limits and $m = (c-a)/(b-a)$ denote location of the mode. Non-negativity of demand ensures $a \geq 0$. We assume $a < c < b$ and strict monotony of F in $[a, b]$. Then $r \in [0, 1)$, $m \in (0, 1)$, and $\theta \in (0, 1)$. Note that every unimodal demand distribution can be "covered" by varying r, m, and θ in their respective ranges. Similarly, every cost structure (i.e., c_o and c_u values) can be covered, if we vary ξ in $(0, 1)$.

In order to derive a lower bound for $\delta_C(\delta_Q)$, first we need to bound F itself. Given a, b, c, and θ, let us define $G \in \mathcal{UD}_{a,b,c,\theta}$ as follows.

$$G(x) = \begin{cases} \dfrac{x-a}{c-a}\theta & \text{if } x \in [a, c) \\ 1 - \dfrac{b-x}{b-c}(1-\theta) & \text{if } x \in [c, b]. \end{cases} \quad (3)$$

See Fig. 1 for a graphical depiction of G. When $\theta = m$, G becomes the uniform distribution in $[a, b]$. We use Q_G^* to denote the optimal order quantity when the demand distribution is G. Then $E_G[C(Q_G^*)]$ denotes the minimum mismatch cost when G is the demand distribution. Using G, Lemma 1 offers a partial bound for every $F \in \mathcal{UD}_{a,b,c,\theta}$.

Lemma 1. *$F(x) \leq G(x)$ if $x < c$ and $F(x) \geq G(x)$ if $x \geq c$ for every $F \in \mathcal{UD}_{a,b,c,\theta}$.*

Proof. We need to focus only on $x \in (a, b)$ as $G(x) = F(x) = 0\ \forall x \leq a$ and $G(x) = F(x) = 1\ \forall x \geq b$. Due to convexity of F in $[a, c]$, $F(\lambda a + (1-\lambda)c) \leq \lambda F(a) + (1-\lambda)F(c) = (1-\lambda)\theta\ \forall \lambda \in (0, 1)$. Replacing $\lambda a + (1-\lambda)c$ by x, $F(x) \leq \{(x-a)/(c-a)\}\theta = G(x)\ \forall x \in (a, c)$. Similarly, due to concavity of F in $[c, b]$, $F(\lambda c + (1-\lambda)b) \geq \lambda F(c) + (1-\lambda)F(b) = 1 - \lambda(1-\theta)\ \forall \lambda \in [0, 1)$. Replacing $\lambda c + (1-\lambda)b$ by x, $F(x) \geq 1 - \{(b-x)/(b-c)\}(1-\theta) = G(x)\ \forall x \in [c, b)$. □

Lemma 1 has the following consequence for Q^*, the optimal decision for F.

Corollary 1. *$Q^* \geq Q_G^*$ if $\xi < \theta$ and $Q^* \leq Q_G^*$ if $\xi \geq \theta$ for every $F \in \mathcal{UD}_{a,b,c,\theta}$.*

Proof. If $\xi < \theta$, $Q^* < c$. Then by Lemma 1, $F(Q^*) \leq G(Q^*)$. If, by contradiction, $Q^* < Q_G^*$ for some $\xi < \theta$, $\xi = F(Q^*) \leq G(Q^*) < G(Q_G^*) = \xi$, which is impossible. The strict inequality is due to strict monotony of G in $[a, b]$. Hence, $Q^* \geq Q_G^*$ if $\xi < \theta$. Similarly, if $\xi \geq \theta$, $Q_G^* \geq c$. Then by Lemma 1, $F(Q_G^*) \geq G(Q_G^*)$. Again, by contradiction, if $Q^* > Q_G^*$ for some $\xi \geq \theta$, $\xi = G(Q_G^*) \leq F(Q_G^*) < F(Q^*) = \xi$, which is impossible. The strict inequality is due to strict monotony of F in $[a, b]$, which we assumed earlier. Hence, $Q^* \leq Q_G^*$ if $\xi \geq \theta$. □

Lemma 1 and Corollary 1 are useful in establishing the lower bound for $\delta_C(\delta_Q)$.

2.2. Lower bound for the numerator

Let us assume that the suboptimal decision, $Q \in [a, b]$. If a' and b' denote the observed lowest and highest demand, then the newsvendor is unlikely to order a quantity that is outside $[a', b']$. Since $[a', b'] \subseteq [a, b]$, we can assume that $Q \in [a, b]$.