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Simulation of ductile tearing by the BEM with 2D domain discretization and a local damage model

Gaëtan Hello [a,*], Hocine Kebir [a], Alain Rassineux [a], Laurent Chambon [b]

[a] Université de Technologie de Compiègne, Laboratoire Roberval CNRS UMR 6253, BP 20529, 60205 Compiègne cedex, France
[b] EADS IW, 18 rue Marius Terce, 31025 Toulouse cedex 03, France

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ABSTRACT

A numerical procedure based on the Boundary Element Method with internal cells and dedicated to the simulation of the ductile tearing of thin metal sheets is presented. Plasticity is handled with an integral formulation based on the initial strain approach involving a discretization of the planar domain. Time integration is performed in an implicit way for the local strain–stress relationships while the global algorithm relies on an explicit formulation. Damage is represented by the scalar parameter of the uncoupled local damage model of Rice and Tracey. Within the scope of our applications, the cracks propagate along paths a priori known. As damage spreads, boundary elements are gradually released. Elastoplastic problems with large yielding zones are solved and compared to reference solutions. At last, the ductile tearing of a specimen is addressed. The calibration of the critical damage parameter leads to numerical results in good agreement with the experimental ones.

1. Introduction

Fracture phenomena occurring in aeronautical structures have to be handled efficiently so as to ensure the structural integrity of the products all along their lifetime. For civil aircrafts, the risks associated to fracture are assessed through the damage-tolerance approach. Indeed, aeronautical panels made of aluminium alloys are able to withstand a stable crack propagation process before the occurrence of the final unsteady fracture. At the crack-tip, the competition between irreversible processes of crack extension and plasticity tends to slow down the propagation. Hence, the accurate simulation of this phenomenon requires a numerical method able to deal effectively with cracks and plasticity simultaneously. The BEM has proven its ability to handle both as summed up by Brebbia et al. [1], Bonnet [2] and Aliabadi [3].

However, the application of the BEM to nonlinear fracture mechanics or damage mechanics has only been treated in a few papers. Herding and Kuhn [4] have developed a BEM formulation associated to Lemaître and Gurson models. Cerrolaza and García [5] have solved geotechnical problems with a procedure including the Mazars model. A non-local continuum damage mechanics model has been presented for the first time by García et al. [6] for the BEM framework. Hatzigeorgiou and Beskos [7] have proposed a method to include the damage influence into the inelastic part of the equations. Lin et al. [8] have dealt with non-local strain softening with a yield limit influenced by damage. An integral operator for non-local strain softening has been described by Sládek et al. [9]. Botta et al. [10] have modeled the damage with a non-local model suitable for concrete and derived the associated Consistent Tangent Operators (CTO). Gun has translated the creep damage effects into the inelastic part of the equations [11] and extended his formulation so that both the creep and the plasticity are taken into account [12]. Benallal et al. [13] have extended their previous works [10] with an emphasis on localization and mesh-independency.

In the present work, a BEM procedure for the resolution of bidimensional problems of ductile tearing is described. The plasticity is managed through the integral formulation based on the initial strain approach. The discretization of the integral equations leads to algebraic systems that provide the displacements and tractions on the boundary as well as the stresses within the domain for fixed plastic strain states. An explicit algorithm combines these systems with an implicit local integration scheme for the plasticity in order to solve efficiently general elastoplastic problems. The uncoupled local damage model of Rice and Tracey [14,15] is employed and describes the damage as a scalar field. This damage parameter is updated as the applied load increases and is used to drive the crack propagation. The extension of the crack is handled with the modification of boundary conditions on specimens where the propagation path is a priori known. Elastoplastic problems are solved with the developed

* Corresponding author.
E-mail addresses: gaetanhello@gmail.com (G. Hello), hocine.kebir@utc.fr (H. Kebir), alain.rassineux@utc.fr (A. Rassineux), laurent.chambon@eads.net (L. Chambon).

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algorithms and the results compared with Finite Element Method (FEM) solutions. Finally, an experiment of ductile tearing on a Center Crack Tension (CCT) specimen is simulated. A proper calibration of the critical damage value provides results in good accordance with experimental ones.

2. Boundary element formulation

2.1. Boundary integral equations

In the BEM formulation for problems involving material nonlinearities, domain integrals are added to the usual boundary integrals. The following equations are expressed in terms of rate so as to comply with the time dependent plasticity description. We consider a domain Ω bounded by $\partial\Omega$ submitted to a given plastic strain rate state $\dot{\varepsilon}^p_{jk}$, the displacement of a point $\mathbf{x}$ can be expressed as

$$c_{ij}(\mathbf{x})\dot{u}_j(\mathbf{x}) = \int_{\partial\Omega} U_{ij}(\mathbf{x},\mathbf{y})\dot{t}_j(\mathbf{y})\,dS(\mathbf{y}) - \fint_{\partial\Omega} T_{ij}(\mathbf{x},\mathbf{y})\dot{u}_j(\mathbf{y})\,dS(\mathbf{y}) + \int_{\Omega} \sigma_{ijk}(\mathbf{x},\mathbf{y})\dot{\varepsilon}^p_{jk}(\mathbf{y})\,d\Omega(\mathbf{y}) \tag{1}$$

where $c_{ij}(\mathbf{x})$ is a free term depending on the geometry around $\mathbf{x}$, $\dot{u}_j$ and $\dot{t}_j$ are respectively the displacement and traction rates on $\partial\Omega$, U_{ij} (Eq. (17)), T_{ij} (Eq. (18)) and σ_{ijk} (Eq. (19)) are kernels of the Kelvin fundamental solution, $\fint$ denotes an integral in the sense of the Cauchy Principal Value (CPV).

For a given point $\mathbf{x}$ inside Ω, the small strain rate $\dot{\varepsilon}_{ij}(\mathbf{x})$ can be derived from Eq. (1) with

$$\dot{\varepsilon}_{ij}(\mathbf{x}) = \frac{1}{2}\left(\frac{\partial\dot{u}_i(\mathbf{x})}{\partial x_j(\mathbf{x})} + \frac{\partial\dot{u}_j(\mathbf{x})}{\partial x_i(\mathbf{x})}\right) \tag{2}$$

The stress rate at point $\mathbf{x}$ can be thereafter obtained from the combination of Eq. (2) with the elastic constitutive Hooke tensor H_{ijkl}

$$\dot{\sigma}_{ij}(\mathbf{x}) = H_{ijkl}(\dot{\varepsilon}_{kl}(\mathbf{x}) - \dot{\varepsilon}^p_{kl}(\mathbf{x})) \tag{3}$$

After some manipulations (see Brebbia et al. [1] for a thorough description of the process), the final expression of the integral equation for internal stresses comes out

$$\dot{\sigma}_{ij}(\mathbf{x}) = \int_{\partial\Omega} D_{ijk}(\mathbf{x},\mathbf{y})\dot{t}_k(\mathbf{y})\,dS(\mathbf{y}) - \int_{\partial\Omega} S_{ijk}(\mathbf{x},\mathbf{y})\dot{u}_k(\mathbf{y})\,dS(\mathbf{y}) + \fint_{\Omega} \Sigma_{ijkl}(\mathbf{x},\mathbf{y})\dot{\varepsilon}^p_{kl}(\mathbf{y})\,d\Omega(\mathbf{y}) + f_{ij}[\dot{\varepsilon}^p_{kl}(\mathbf{x})] \tag{4}$$

with D_{ijk}, S_{ijk} and Σ_{ijkl} kernels derived from those of Eq. (1) and defined in Eqs. (20)–(22), f_{ij} (Eq. (23)) a free term, whose definition has first been given by Bui [16], depending on the plastic strain rate at the source point $\mathbf{x}$.

2.2. Numerical discretization

The integral equations (1) and (4) provide continuous representations of both boundary displacement and domain stress rates. When it comes to problems on domains with general shapes, the solution has to rely on numerical procedures. The domain and the continuous equations have therefore to be discretized. The discretization process first concerns the geometry. The boundary $\partial\Omega$ is divided into $n_{\partial\Omega}$ segments. The domain Ω is decomposed into n_Ω triangular cells. Then, unknown mechanical fields on the elementary supports are expressed in terms of polynomial functions and nodal unknowns. A local quadratic discontinuous approximation is chosen for the boundary fields. For a given boundary element $\partial\Omega_e$, the positions of the interpolation points depend on a scalar parameter α (Fig. 1). A value of 1/6 has proven its reliability.

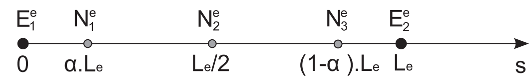

Fig. 1. Parametrization of the collocation points on the boundary element $\partial\Omega_e$.

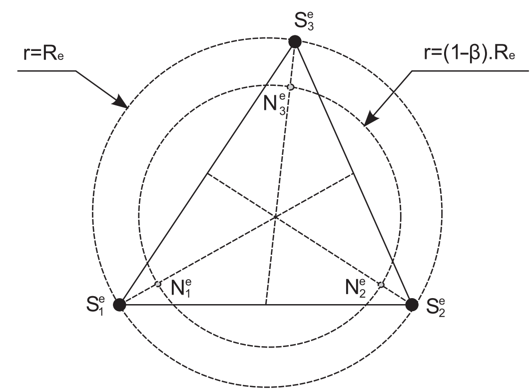

Fig. 2. Parametrization of the collocation points on the domain element Ω_e.

Concerning the local plastic field in the triangular cells, a linear discontinuous approximation is used. The interpolation points are positioned within the triangles with a parameter β (Fig. 2). The choice of $\beta = 1/3$ leads to both stability and accuracy during the simulations.

The need for discontinuous interpolations on both boundary and domain elements is justified by the application to fracture problems. Cracks propagation is modeled through the progressive release of boundary elements and the update of boundary-conditions. Since successive crack-tips are located at boundary elements extremities, collocation points must then be located away from these extremities so as to ensure the finiteness of tractions, stresses and plastic stains. Strictly speaking, discontinuous boundary elements and semi-discontinuous domain elements are only mandatory along cracks paths. However, for the sake of programming simplicity and at the expense of computational cost, all elements are here formulated with discontinuous interpolation schemes.

Since the integration supports and the elementary unknown fields have been discretized, the continuous equation (1) can be transformed in

$$\frac{1}{2}\dot{u}_i(\mathbf{x}) = \sum_{e=1}^{n_{\partial\Omega}} \int_{\partial\Omega_e} U_{ij}(\mathbf{x},\mathbf{y})\left(\sum_{m=1}^{P_{1D}} N^{e,m}_{1D}(\mathbf{y})\dot{t}^{e,m}_j\right) dS(\mathbf{y}) - \sum_{e=1}^{n_{\partial\Omega}} \fint_{\partial\Omega_e} T_{ij}(\mathbf{x},\mathbf{y})\left(\sum_{m=1}^{P_{1D}} N^{e,m}_{1D}(\mathbf{y})\dot{u}^{e,m}_j\right) dS(\mathbf{y}) + \sum_{e=1}^{n_\Omega} \int_{\Omega_e} \sigma_{ijk}(\mathbf{x},\mathbf{y})\left(\sum_{m=1}^{P_{2D}} N^{e,m}_{2D}(\mathbf{y})\dot{\varepsilon}^{p\ e,m}_{jk}\right) d\Omega(\mathbf{y}) \tag{5}$$

where 1/2 is the value of $c_{ij}(\mathbf{x})$ when $\mathbf{x}\in\partial\Omega$ and $\partial\Omega$ is smooth enough at $\mathbf{x}$, $N^{e,m}_{1D}$ is the mth interpolation function on $\partial\Omega_e$, $N^{e,m}_{2D}$ is the mth interpolation function on Ω_e, $\dot{t}^{e,m}_j$, $\dot{u}^{e,m}_j$ and $\dot{\varepsilon}^{p\ e,m}_{jk}$ are the discrete unknowns.

The discretization process applied to Eq. (4) gives

$$\dot{\sigma}_{ij}(\mathbf{x}) = \sum_{e=1}^{n_{\partial\Omega}} \int_{\partial\Omega_e} D_{ijk}(\mathbf{x},\mathbf{y})\left(\sum_{m=1}^{P_{1D}} N^{e,m}_{1D}(\mathbf{y})\dot{t}^{e,m}_k\right) dS(\mathbf{y}) - \sum_{e=1}^{n_{\partial\Omega}} \int_{\partial\Omega_e} S_{ijk}(\mathbf{x},\mathbf{y})\left(\sum_{m=1}^{P_{1D}} N^{e,m}_{1D}(\mathbf{y})\dot{u}^{e,m}_k\right) dS(\mathbf{y}) + \sum_{e=1}^{n_\Omega} \fint_{\Omega_e} \Sigma_{ijkl}(\mathbf{x},\mathbf{y})\left(\sum_{m=1}^{P_{2D}} N^{e,m}_{2D}(\mathbf{y})\dot{\varepsilon}^{pe,m}_{kl}\right) d\Omega(\mathbf{y}) + f_{ij}(\dot{\varepsilon}^p_{kl}(\mathbf{x})) \tag{6}$$