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# The method of fundamental solutions for acoustic wave scattering by a single and a periodic array of poroelastic scatterers

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The method of fundamental solutions (MFS) is now a well-established technique that has proved to be reliable for a specific range of wave problems such as the scattering of acoustic and elastic waves by obstacles and inclusions of regular shapes. The goal of this study is to show that the technique can be extended to solve transmission problems whereby an incident acoustic pressure wave impinges on a poroelastic material of finite dimension. For homogeneous and isotropic materials, the wave equations for the fluid phase and solid phase displacements can be decoupled thanks to the Helmholtz decomposition. This allows for a simple and systematic way to construct fundamental solutions for describing the wave displacement field in the material. The efficiency of the technique relies on choosing an appropriate set of fundamental solutions as well as properly imposing the transmission conditions at the air–porous interface. In this paper, we address this issue showing results involving bidimensional scatterers of various shapes. In particular, it is shown that reliable error indicators can be used to assess the quality of the results. Comparisons with results computed using a mixed pressure–displacement finite element formulation illustrate the great advantages of the MFS both in terms of computational resources and mesh preparation. The extension of the method for dealing with the scattering by an infinite array of periodic scatterers is also presented.



## 1. Introduction

Poroelastic materials are often used for their good sound absorbing capabilities in the middle and high frequency range. Typical applications can be found in the context of the transport noise reduction or to enhance the quality of room acoustics. The description of wave propagation in porous media is not limited to audible acoustics as Biot's model [8] was originally developed for geological applications. Because of their inherent diphasic features and the strong contrasts that may exist between the solid and the fluid phases, wave propagation modeling remains a difficult task often leading to heavy computational costs. In the context of the finite element method (FEM), some developments have been proposed using Lagrange or hierarchical finite elements [5,29,18]. Because of the scale disparity, the so-called poroelastic elements have a slower convergence rate than purely elastic or fluid elements [29]. To make matters worse, Biot's equations are frequency dependent and large FEM system matrices have to be recalculated for each frequency. For homogeneous and isotropic materials, the boundary element method (BEM) offers an alternative [31]. The method has the advantage of reducing the entire problem to one with only unknowns on the boundaries. However, the system matrix is full and there is still the need to discretize the boundary surface as well as performing regular and singular integrations over each boundary element.

In the past decade, several researchers have focused their work on meshless methods in order to avoid the time-consuming mesh generation process for complicated geometries. In this regard, the method of fundamental solution (MFS) has been shown to be efficient for solving a large variety of physical problems as long as a fundamental solution of the underlying differential equation(s) is known. In particular, the MFS is suitable for scattering problems by choosing appropriate fundamental solutions satisfying the radiation condition at infinity. The method shares the same advantages as the BEM over domain discretization methods because there is no need to create a mesh over the entire domain. Furthermore, as no integration is needed, some numerical difficulties encountered with the BEM are avoided. For comprehensive reviews on applications of the MFS for scattering and radiation problems one can refer to Fairweather et al. [13,14].

In this work, we are interested in applying as well as assessing the MFS for the numerical simulation of a bidimensional incident acoustic wave scattered by a poroelastic material. To the authors' knowledge, such problems have never been addressed using the MFS and although analytical solutions are available for canonical

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geometries such as cylindrical and spherical scatterers [15,16], there is a need for fast and accurate methods taking into account scatterers of arbitrary shape. The applications we have in mind range from multiple scattering, the modeling of double porosity materials [6,26] to shape foam optimization modeling [19,30].

In a preliminary note presented by the authors [25], the application of the MFS to such problems was briefly presented. In this work, all the details of the implementation of the method and the investigation of its accuracy and numerical characteristics, including the condition number of the resulting matrices are given. The present paper is organized as follows. After presenting the MFS formulation in Section 2, the method's performance is measured with respect to an academic problem, the circular-shaped scatterer, for which analytical solutions are available. In particular, it is shown that reliable error indicators can be used to assess the quality of the results. In Section 4, various numerical examples of increasing difficulty are presented with a comparison to FEM results. The last section shows applications concerning the scattering of an incident plane wave by an infinite periodic array of porous structures.

## 2. Formulation of the method

Consider a time-harmonic acoustic plane wave $\varphi_0^{\rm inc} = A^{\rm inc} \exp {\rm i}k_0\zeta\cdot\mathbf{x}$ (with the convention ${\rm e}^{-{\rm i}\omega t}$) in an unbounded exterior domain $\Omega^e$ incident upon one (or more) poroelastic acoustic foam(s) denoted by $\Omega^i$ with boundary $\Gamma$ as shown in Fig. 1. We denote by $\mathbf{n}$, the unit outward normal vector to $\Omega^i$. In the surrounding acoustic domain $\Omega^e$, the fluid is inviscid and the acoustic displacement potential $\varphi_0$ obeys the wave equation

$$\Delta\varphi_0 + k_0^2\varphi_0 = 0. \tag{1}$$

Here, $k_0 = \omega/c_0$ is the classical wavenumber defined as the ratio of the angular frequency $\omega$ and the sound speed $c_0$. To express the transmission conditions at the interface $\Gamma$, it is convenient to introduce the particle displacement perturbation $\mathbf{w} = \nabla\varphi_0$. With this definition, the acoustic pressure is obtained from the linearized momentum equation as $p = \rho_0\omega^2\varphi_0$. In (1), it is natural to split the potential into an incident and a scattered part as $\varphi_0 = \varphi_0^{\rm inc} + \varphi_0^{\rm sc}$ (and similarly for the pressure and displacement). Here we require the scattered field to satisfy the usual Sommerfeld radiation condition at infinity. In the poroelastic medium the acoustic waves propagation is described by Biot's model [8]. This latter is based on the superposition of a fluid phase and a solid phase which are coupled together and respectively described by the fluid phase displacement $\mathbf{U}$ and the solid phase displacement $\mathbf{u}$. For the time-harmonic representation, we have the following coupled system [8]:

$$\nabla\cdot\boldsymbol{\sigma}^s + \omega^2(\rho_{11}\mathbf{u} + \rho_{12}\mathbf{U}) = 0, \tag{2}$$

$$\nabla\cdot\boldsymbol{\sigma}^f + \omega^2(\rho_{12}\mathbf{u} + \rho_{22}\mathbf{U}) = 0. \tag{3}$$

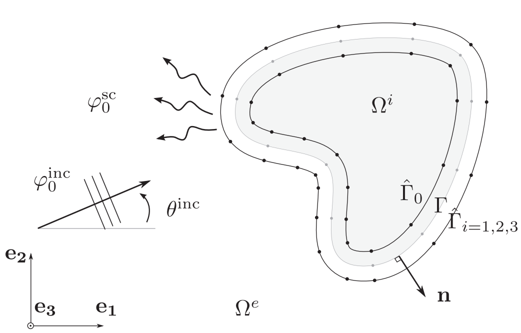


**Fig. 1.** Scattering geometry.

The solid and fluid phase stress tensors are given by

$$\boldsymbol{\sigma}^s = (A\nabla\cdot\mathbf{u} + Q\nabla\cdot\mathbf{U})\mathbf{I} + 2N\boldsymbol{\varepsilon}^s, \tag{4a}$$

$$\boldsymbol{\sigma}^f = (Q\nabla\cdot\mathbf{u} + R\nabla\cdot\mathbf{U})\mathbf{I}, \tag{4b}$$

where $\boldsymbol{\varepsilon}^s = 1/2(\nabla\mathbf{u} + (\nabla\mathbf{u})^{\rm t})$ is the usual strain tensor and $\mathbf{I}$ is the identity matrix. The total stress tensor $\boldsymbol{\sigma}^t$ is, by definition, the sum of $\boldsymbol{\sigma}^f$ and $\boldsymbol{\sigma}^s$. Biot's coefficients $A,N,Q,R$ are related to the material properties by the Allard–Johnson model. Their expressions can be found in the literature or in the reference textbook [2] as well as the other quantities introduced in this section. These quantities are all complex and frequency-dependent, $A$ and $N$ correspond to the Lamé coefficients, $R$ is the effective bulk modulus of the fluid phase and $Q$ indicates the coupling of the two phases volumic dilatation. The imaginary parts of $A$ and $N$ include the structural damping and, in $Q$ and $R$ these parts include the thermal dissipation. The imaginary parts of the effective density coefficients $\rho_{11}$, $\rho_{22}$ and $\rho_{12}$ take into account viscous damping. The complete solution to the problem is found after applying the classical air–porous transmission conditions [11,22] on the interface $\Gamma$, i.e.

$$p_p - p^{\rm sc} = p^{\rm inc}, \tag{5a}$$

$$\phi\mathbf{U}\cdot\mathbf{n} + (1-\phi)\mathbf{u}\cdot\mathbf{n} - \mathbf{w}^{\rm sc}\cdot\mathbf{n} = \mathbf{w}^{\rm inc}\cdot\mathbf{n}, \tag{5b}$$

$$\boldsymbol{\sigma}^t\mathbf{n} + p^{\rm sc}\mathbf{n} = -p^{\rm inc}\mathbf{n}. \tag{5c}$$

Here $\phi$ is the porosity and the pore pressure $p_p$ is obtained from the fluid phase tensor as $-\mathbf{I}\phi p_p = \boldsymbol{\sigma}^f$.

For homogeneous and isotropic materials, the wave equation for the fluid phase and solid phase displacements can be decoupled thanks to the Helmholtz decomposition. Both solid and fluid displacement fields are then written as

$$\mathbf{u} = \nabla\varphi + \nabla\wedge(\varphi_3\cdot\mathbf{e}_3) \quad \text{and} \quad \mathbf{U} = \nabla\chi + \nabla\wedge(\Theta\cdot\mathbf{e}_3). \tag{6}$$

After decoupling the equations, we have [2]: $\varphi = \varphi_1 + \varphi_2$ and $\chi = \mu_1\varphi_1 + \mu_2\varphi_2$ where

$$\mu_i = \frac{Pk_i^2 - \omega^2\rho_{11}}{\omega^2\rho_{12} - Qk_i^2}, \quad i = 1,2 \tag{7}$$

are the wave amplitude ratios between the two phases in the porous material (here, $P = A+2N$). Similarly, the potential $\Theta$ is simply obtained as $\Theta = \mu_3\varphi_3$ with $\mu_3 = \rho_{12}/\rho_{22}$. Under this form, each potential $\varphi_i$ ($i = 1,2,3$) satisfies the Helmholtz equation

$$\Delta\varphi_i + k_i^2\varphi_i = 0, \tag{8}$$

and the associated complex wavenumbers are

$$k_1^2 = \frac{\omega^2}{2(PR-Q^2)}(P\rho_{22} + R\rho_{11} - 2Q\rho_{12} + \sqrt{D}), \tag{9}$$

$$k_2^2 = \frac{\omega^2}{2(PR-Q^2)}(P\rho_{22} + R\rho_{11} - 2Q\rho_{12} - \sqrt{D}), \tag{10}$$

$$k_3^2 = \frac{\omega^2}{N}\left(\frac{\rho_{11}\rho_{22} - \rho_{12}^2}{\rho_{22}}\right). \tag{11}$$

Here, $D$ stands for the discriminant of a quadratic equation and $D = (P\rho_{22} + R\rho_{11} - 2Q\rho_{12})^2 - 4(PR-Q^2)(\rho_{11}\rho_{22} - \rho_{12}^2)$. Physically, there are two compressional waves associated with $\varphi_1,\varphi_2$ and one rotational (shear) wave associated with $\varphi_3$. They all propagate in the two phases and their relative contributions are given by the coefficients $\mu_i$. If such a decomposition holds in elastodynamics, the coexistence of two phases in the poroelastic media adds another fluid-borne compressional wave which is not present is elastic solids.

The MFS implementation starts by choosing an appropriate set of fundamental solutions for both propagative domains $\Omega^e$ and $\Omega^i$.