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Boundary element method for 2D solids with fluid-filled pores

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ABSTRACT

In this paper, a boundary element method is developed for solving the problems of 2D solids with fluid-filled pores. The solid is assumed as linear elastic, which contains many fluid-filled pores of various shapes, and the fluid filling the pores is assumed to be linear compressible. Two different approaches, named superposition method and multi-subdomain method have been presented. The first one is based on the principle of superposition, in which all the pressures in the fluid-filled pores will be determined first, and then all the other boundary unknowns can be computed. In the other approach, the subdomains of the fluid in pores are solved to obtain the relation of the interface displacements with the interface pressure first, and then all the boundary unknowns, including the fluid pressure in each pore, can be solved simultaneously. Two simple examples of the 2D solids containing one circular fluid-filled pore are applied to verify the accuracy and to show the efficiency of the presented methods. And then, the effective elastic modulus and effective Poisson's ratio are simulated based on several models of the 2D solids containing 100 randomly distributed circular or elliptical fluid-filled pores. The numerical results computed by the two schemes have nearly the same accuracy, whereas the multi-subdomain method has higher computational efficiency than the superposition method. Some differences between the results obtained by the BEM and those given by Kachanov's method in the literature have been observed, which will be further investigated in the future work.

1. Introduction

There are a variety of porous media in the nature, such as rocks, soil, polymer foams, metal foams, concrete foams, sponges and biological tissues, some of which are fully saturated, some are partially saturated and some are dry ones. In most porous media, the fluid in pores can flow from one pore to the others. The fluid in different pores is isolated in some cases of the porous media, which can be modelled as solid with numerous fluid-filled pores. The mechanical properties of such porous media will be affected to a certain extent by the shapes, sizes and microstructures of the pores. The investigation on the effective elastic properties of such porous media has attracted considerable interests in the research of solid mechanics.

O'Connell and Budiansky [1,2] investigated the effective elastic properties of materials with fluid-filled pores in the special case of pores' geometry—narrow, crack-like cavities, whereas the applicability of their results is limited by the implicit assumption that all cavities have the same aspect ratios. The polarization phenomenon of fluid pressure induced by the applied load was addressed by Zimmerman [3], assuming that the porous space is interconnected together. Kachanov [4] considered an arbitrary orientational distribution of narrow crack-like cavities and examined the fluid pressure polarization as well as the impact of fluid on stress interactions on cracks. Shafiro and Kachanov [5,6] presented a general 3D analysis that covers fluid-filled pores of arbitrary ellipsoidal shapes, in particular, mixtures of cavities of diverse shapes, including pores and cracks. Besides, Giraud and Huynh [7] applied the Eshelby tensor to determine the effective poroelastic properties of anisotropic rocks-like composites.

Although much research has been done on the effective elastic properties of fluid-saturated porous media, most of them are based on the analytical methods, in which some specific assumptions or limitations usually had to be adopted. Only few research works have been reported to examine the effective elastic properties of materials with fluid-filled pores by numerical methods, especially by the boundary element method. Since the boundary element method only needs the boundary discretization, it has obvious advantages over other numerical methods for various elastic problems containing numerous cracks or inclusions [8–13]. The main purpose of this paper is to develop the boundary element method for the problems of 2D solid with fluid-filled pores, and to simulate the equivalent mechanic behaviors of such materials.

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2. Formulation of 2D solids with fluid-filled pores

2.1. Description of the problem

A 2D solid containing n randomly distributed fluid-filled pores of arbitrary shapes is considered as shown in Fig. 1, where Ω and Γ denote the solid domain and its outer boundary, $\Omega_1, \Omega_2, \ldots, \Omega_i, \ldots, \Omega_n$ denote the fluid-filled pores, and $\Gamma_1, \Gamma_2, \ldots, \Gamma_i, \ldots, \Gamma_n$ denote their boundaries, which can also be considered as the interfaces of the solid and fluid, whereas Γ_t and Γ_u indicate the given traction part and the given displacement part of the outer boundary, respectively.

When the external loads are applied on the boundary Γ, the deformation of the solid and the pressure of the fluid in pores will interact with each other on their interfaces, thus this problem can be regarded as a typical fluid–solid interaction problem. In this paper, the fluid-filled in the pores is assumed to be linear nonviscous and compressible. The fluid inside the i-th pore satisfies

$$-\Delta S_i/S^{(i)} = \kappa^{(i)} p_i \quad (i=1,\ldots,n) \tag{1}$$

where $S^{(i)}$, ΔS_i, $\kappa^{(i)}$ and p_i stand for the area, the area variation, the compressibility and the pressure variation of the fluid filled in the i-th pore, respectively.

If the fluid inside the i-th pore is incompressible, i.e. $\kappa^{(i)}=0$, the area of each pore will keep constant, but its shape will be changeable.

It should be mentioned that, although the fluid pressure, p_i, in each pore is constant, they are different for different pores. Therefore, how to determine all these pressures in the fluid-filled pores is of most importance for solving such problems. Once the pressures in the fluid-filled pores are determined, the problem is transformed into a typical boundary value problem of elastic solid, which can be solved easily.

2.2. Formulation of the BEM for the 2D elastic solid [14,15]

It is well-known that the boundary integral equation for the elastic problem can be derived from the Betti's reciprocal theorem and Kelvin's solution. The Somigliana's identity for the linear elastic plane strain problem in a multiply connected region, as shown in Fig. 1 can be written as

$$u_\alpha(P) = \int_{\Gamma+\sum\Gamma_i} U^*_{\alpha\beta}(P;q)t_\beta(q)d\Gamma(q) - \int_{\Gamma+\sum\Gamma_i} T^*_{\alpha\beta}(P;q)u_\beta(q)d\Gamma(q) \tag{2}$$

where P and q stand for the source point in the domain and the field point on the boundary, $u_\beta(q)$, $t_\beta(q)$ and $u_\alpha(P)$ denote the displacements and tractions at the boundary point q and the displacement at the source point P, $U^*_{\alpha\beta}(P;q)$ and $T^*_{\alpha\beta}(P;q)$ are the displacement and traction fundamental solution, respectively. For the plane strain problem, the Kelvin's fundamental solutions can be written as

$$U^*_{\alpha\beta}(P;q) = \frac{1}{8\pi G(1-\nu)}\left[(3-4\nu)\delta_{\alpha\beta}\ln\frac{1}{r} + r_{,\alpha}r_{,\beta}\right] \tag{3}$$

$$T^*_{\alpha\beta}(P;q) = -\frac{1}{4\pi(1-\nu)r}\left\{\frac{\partial r}{\partial n}\left[(1-2\nu)\delta_{\alpha\beta} + 2r_{,\alpha}r_{,\beta}\right] - (1-2\nu)(r_{,\alpha}n_\beta - r_{,\beta}n_\alpha)\right\} \tag{4}$$

where r is the distance from the source point P to the field point q.

As the source point P approaches to the boundary point p, the well-known boundary integral equation can be derived as

$$c_{\alpha\beta}(p)u_\beta(p) = \int_{\Gamma+\sum\Gamma_i} U^*_{\alpha\beta}(p;q)t_\beta(q)d\Gamma(q) - \int_{\Gamma+\sum\Gamma_i} T^*_{\alpha\beta}(p;q)u_\beta(q)d\Gamma(q) \tag{5}$$

where $c_{\alpha\beta}$ is a free term, which depends on the geometry of boundary at the source point p, and if the boundary is smooth at the point p, then $c_{\alpha\beta}=1/2\delta_{\alpha\beta}$.

In the procedure of boundary element method, the boundary Γ of the domain Ω, together with the inner boundary Γ_i, should be discretized into boundary elements, the boundary variables are discretized, and correspondingly the boundary integral equations are discretized into a system of algebraic equations, which can be written in the matrix form, namely

$$\boldsymbol{Hu} = \boldsymbol{Gt} \tag{6}$$

where $\boldsymbol{u}$ and $\boldsymbol{t}$ are the vectors containing all the nodal values of displacements and tractions, respectively, $\boldsymbol{H}$ and $\boldsymbol{G}$ are the corresponding coefficient matrices, which are obtained by the integration of the product of fundamental solution and shape function over the boundary elements. By using the boundary conditions and shifting the unknown variables to the left hand side and the known ones to the right hand side, the resulted equation system is a typical system of algebraic equation

$$\boldsymbol{Ax} = \boldsymbol{c} \tag{7}$$

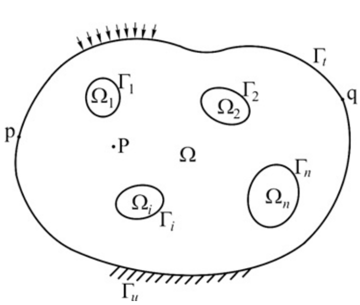

Fig. 1. Model of a 2D solid with fluid-filled pores.

3. Two approaches for solving 2D solids with fluid-filled pores

In this section, two approaches are presented for solving the 2D solid with fluid-filled pores. The first approach deals with the solids of multiply connected domain under the unknown fluid pressure on the inner boundary of pores, the deformations resulted by external load and the unit fluid pressure subjected at each individual pore are solved independently, and then superposed to satisfy the constitutive law of the fluid (Eq. (1)) in each pore, and in this way the fluid pressures are determined first, and then the final solution can be found. Therefore, it can be named as the superposition method. Another approach deals with a multi-subdomain problem, one multiply connected subdomain of solid and n subdomains of fluid inclusion. The numerical method for the simulation of solids with multi-inclusions is generalized to solve such problems. The variables of the fluid subdomain are condensed into the interface conditions of the solid subdomain. Then the final solution can be found directly by solving the BE equations of the multiply connected solid subdomain. This approach can be named as multi-subdomain method.

3.1. Superposition method

In this approach, it is dealt with the solid of multiply connected domain under the unknown fluid pressure on the