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DRBEM solution of the thermo-solutal buoyancy induced mixed convection flow problems

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ABSTRACT

Mixed convective heat and solutal transport is important in engineering applications such as nuclear waste disposal, crystal growth and oceanography. Mixed convective in a lid-driven cavity and through channels with backward-facing step is studied by solving the equations of conservation of mass, momentum, energy and solutal concentration numerically. The governing equations are solved by using dual reciprocity boundary element method (DRBEM) with constant elements in terms of stream function, vorticity, temperature and concentration. Vorticity, energy and concentration equations are transformed to the form of modified Helmholtz equations by utilizing forward difference with relaxation parameters for the time derivatives, and also approximating Laplacian terms at two consecutive time levels. The DRBEM application is carried out with the fundamental solution of modified Helmholtz equation, resulting in linear algebraic systems for the time dependent unknowns. Inhomogeneities in modified Helmholtz equations are approximated with the thin plated radial basis functions. Computations are carried out for several values of Richardson number, buoyancy ratio and Reynolds number. When the temperature and solutal concentration boundary conditions are changed, the thermal and solutal buoyancy forces can oppose or aid each other. The effects of these parameters on the flow behavior and heat transfer are shown in terms of graphics.

1. Introduction

Mixed convective heat and solutal transport play an important role in many engineering applications such as plasma spray coating, nuclear waste disposal, crystal growth and oceanography. It is reported in [1], that "In applications such as nuclear waste disposal facilities, the primary interest will be the study of thermal energy transport in the presence of solute transport whereas the study of solute transport in the presence of heat transfer becomes important in the analysis of crystal growth, plasma spray coating problems".

In simulation of a real problem of continuous removal of pollutants generated at the bottom wall of a cavity by the moving top lid, the interaction between fluid inertial force and thermo-solutal buoyancy forces on convective heat and mass transfer becomes important. Al-Amiri et al. [2] numerically investigated steady mixed convection in a square cavity under the combined buoyancy effects of thermal and mass diffusion by using Galerkin's weighted residual finite element method. It is the first study about the combined heat and mass transfer by mixed convection which was defined in a lid-driven cavity flow. They give the range where high heat and mass transfer rates can be obtained for a given Richardson number. Kumar et al. [1] examined deeply the interaction between fluid inertial force and thermo-solutal buoyancy forces on convective heat transfer in a lid-driven square cavity by using various values of Ri, Re and N. The velocity–vorticity form of Navier–Stokes equations was used for the analysis of the double-diffusive mixed convection in a lid-driven cavity with a heated blockage at the bottom wall of the cavity in [3,4]. In these studies Galerkin's weighted residual finite element method was used as a numerical method. The results show the influence of obstruction on the heat and mass transfer inside the enclosure. Alleborn et al. [5] were concerned with steady two-dimensional flow accompanied by heat and mass transfer together with species concentration in a shallow lid-driven cavity with a moving heated bottom lid, and a cooled top lid moving with a different constant velocity. In their study an analytical solution, for different cavity orientation and limiting values of parameters, is given. Maiti et al. [6] presented a numerical study about heat and mass transfer in a square cavity with a sliding top lid in the presence of vertical temperature and concentration gradients. The influence of solutal and thermal buoyancy forces on the flow, and the effect of shearing of the top lid are analyzed by different situations. Deng et al. [7] numerically studied laminar double-diffusive mixed convection in a two-dimensional displacement ventilated enclosure with discrete heat and contaminant sources by investigating the effects of Grashof number, buoyancy ratio, Reynolds number and ventilation mode. Both Maiti et al. [6]

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and Deng et al. [7] used finite volume method to obtain the numerical results in their studies.

In addition, convective heat transfer in flow through channels with backward-facing step (BFS) is important in engineering applications such as heat exchanger devises, electronic cooling and turbine blade cooling. The recirculation region affects the convection heat transfer characteristics of the system. Therefore, it is important to understand the mechanisms of heat transfer. Under the effect of suction and blowing, the reattachment length and the length of the top secondary recirculation bubble are computed by Abu-Nada et al. [8] by using the finite volume method. Velazquez et al. [9] presented the effect that a forced flow pulsation may have on laminar heat transfer enhancement behind a two-dimensional backward facing step. Finite-point formulation which was developed by themselves was used to solve the problem. Brown and Lai [10] investigated the problem of combined heat and mass transfer from the horizontal channel with an open cavity heated on the bottom wall. In the computations, numerical results were obtained by using different values of buoyancy ratio, Lewis number and Reynolds number to obtain the correlations of combined heat and mass transfer by mixed convection from an open cavity in a horizontal channel. In the BFS channel problems both the solutal buoyancy force and the thermal buoyancy force contributions must be taken into consideration on the flow field variation and heat transfer. The density gradients, which are generated by the solutal concentration gradient, develop the solutal buoyancy force. At the channel entry, the fluid inertial forces are dominant but when the fluid moves between the walls, the thermo-solutal buoyancy forces influence the flow field structure. If the convective heat transfer contains the thermo-solutal buoyancy forces, then it is called double diffusive mixed convection. When the temperature and solutal concentration boundary conditions are changed, the thermal and solutal buoyancy forces can oppose or aid each other. There are studies on the changes in recirculatory flow behavior because of the thermo-solutal buoyancy forces in a BFS horizontal channel in [12,13] and Galerkin's finite element method was used to obtain the numerical results.

Dual reciprocity boundary element method (DRBEM) is a numerical solution technique which can treat the nonlinearities in the partial differential equations by taking them as right hand side functions [14]. In DRBEM, only a boundary integral equation is obtained for inhomogeneous, nonlinear or time dependent problems by eliminating the domain integral through the BEM formulation. Bozkaya and Tezer-Sezgin solved the natural convection flow in differentially heated enclosure by using the coupling of the DRBEM in space with the DQM in time [15]. Gümgüm and Tezer-Sezgin [16] solved the unsteady natural convection flow of nanofluids in enclosures with a heat source by using the DRBEM. The time derivative is discretized by using a central finite difference scheme. Both of these studies used the fundamental solution of Laplace equation in DRBEM since all the terms other than Laplacian are treated as inhomogeneity. DRBEM has the advantage of discretizing only the boundary of the problem. Thus, the resulting system of algebraic equations is very small compared to all the other domain discretization methods, such as finite difference method (FDM), finite element method (FEM), and finite volume method (FVM). There is no application of DRBEM on the modified Helmholtz equations for the numerical solutions of thermo-solutal buoyancy-driven flow problems to the best of our knowledge.

In this study, we consider the DRBEM solution of the two-dimensional thermo-solutal buoyancy-driven flow problems. The first problem is the mixed convective heat and solutal transport in a lid-driven cavity, and the second problem is the thermo-solutal stratification in a horizontal channel with backward-facing step. In both problems, stream function, vorticity, temperature and concentration variables are used. The vorticity, energy and concentration equations are transformed to modified Helmholtz equations by utilizing forward difference with relaxation parameters for the time derivatives before discretization in the space direction is performed. This procedure eliminates the need for another time integration scheme in vorticity transport, energy and concentration equations. The resulting modified Helmholtz equations are solved by DRBEM using the fundamental solution $K_0(x)$ whereas in the stream function Poisson's equation $\ln(x)$ is made use of. The inhomogeneities are approximated by using coordinate functions $f=1+r$ in the stream function equation and $f=r^2\log r$ in the vorticity, energy and concentration equations. The missing vorticity boundary conditions are also obtained with the help of the coordinate matrix constructed with the radial basis function $f(r)$ in DRBEM which is another advantage of DRBEM. Alsoy-Akgün and Tezer-Sezgin [17] used DRBEM procedure in the solution of the natural convection and lid-driven flow problems in cavities. The results were obtained for Navier–Stokes equations in a lid-driven cavity with Re values up to 2000, and $Ra=10^6$ was achieved for natural convection flow in cavities. In another study of Alsoy-Akgün and Tezer-Sezgin [18] natural convection in a cavity under a magnetic field was solved by using DRBEM. Results were obtained again up to Ra values of 10^6 and Hartmann number $Ha=300$. In this paper, the results of thermo-solutal buoyancy-driven flow problems in a square cavity and in a horizontal channel with backward-facing step are given for varying values of Ri, Re, N. The solutions are obtained by solving small algebraic systems at a cheap computational cost due to the boundary nature of DRBEM.

2. Governing equations

The thermo-solutal buoyancy-driven flow is governed by the equations that represent conservation equations for mass, momentum, energy and solutal concentration. Together with the velocity–vorticity form of the vorticity transport equation the governing equations are given in a nondimensional form as

$$\begin{aligned}
&\nabla^2\psi=-w\\
&\frac{1}{Re}\nabla^2 w=\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}-Ri\left(\frac{\partial T}{\partial x}+N\frac{\partial C}{\partial x}\right)\\
&\frac{1}{Re\,Pr}\nabla^2 T=\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\\
&\frac{1}{Re\,Sc}\nabla^2 C=\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}
\end{aligned}\tag{1}$$

in which velocity components u, v and vorticity w are defined as

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}\tag{2}$$

and

$$w=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\tag{3}$$

where ψ, w, T, and C are the stream function, vorticity, temperature, and solutal concentration, respectively. Re is the Reynolds number given by $Re=U_0H/v$, where U_0, H and v are characteristic velocity, characteristic length and kinematic viscosity, respectively. Ri is the Richardson number given by Gr_T/Re^2, where Gr_T is the Grashof number due to the thermal diffusion. N is the buoyancy ratio given by $\beta_C\Delta C/\beta_T\Delta T$, where β_C volumetric solutal concentration expansion coefficient, β_T volumetric thermal expansion coefficient, $\Delta C=C_h-C_c$ (C_h and C_c are high and low solutal concentrations) and $\Delta T=T_h-T_c$ (T_h and T_c are high and low temperatures).