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Seepage analysis in multi-domain general anisotropic media by three-dimensional boundary elements

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ABSTRACT

A three-dimensional boundary element solution for the seepage analysis in multi-domain general anisotropic media has been developed based on the transformation approach. Using analytical eigenvalues and eigenvectors of the hydraulic conductivity tensor, a closed-form coordinate transformation matrix has been provided to transform the quadratic form of governing equation of seepage for the general anisotropic media to the Laplace equation. This transformation allows the analysis to be carried out using any standard BEM codes for the potential theory on the transformed space by adding small pre- and post-processing routines. With this transformation, any physical quantity like the total head remains unchanged at corresponding nodes on the physical and transformed space, and the normal gradient across the domain boundaries should also be transformed. In multi-domain problems, compatibility equations (equality of the potential on corresponding nodes on the interface) and equilibrium equations (conservation of the flux across the interface boundaries of adjacent domains) on the corresponding nodes of interface between two neighbor domains are needed for boundary element method. In the transformed space, the compatibility equation remains unchanged. However, due to the distortion of boundaries in the mapped space and therefore misalignment of the unit outward normal vectors along the inter-domain boundaries, the equilibrium of the normal fluxes have to be transformed accordingly. Based on the proposed transformation, the normal to boundary flux boundary conditions in the mapped space and the transformed equilibrium equation for interface of adjacent zones have been given in this paper. Examples have been solved with the proposed scheme and the results were verified with the finite element method. Excellent agreement of the results shows the veracity of the proposed transformation and the formulas given for transformation of equilibrium equation for multi-domain general anisotropic media.

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1. Introduction

The boundary element method (BEM) was developed in the 1960s as an efficient numerical technic [1]. The method then has been used vastly for solving different problems in various fields. The method is of the great interest because of the numerical form of equations are written only on the domain boundaries. This allows for discretizing the just boundaries of the domain and there is no need for the discretization of the domain area. As modeling and discretization of the problem is one of the most costly parts of analyses [2], without the need of dealing with the interior mesh, the BEM is more cost effective in mesh preparation [1]. This is specially the case when working on three-dimensional problems when only surface boundaries are going to be discretized. In comparison with other numerical methods which mandates the discretization of the whole media such as finite element method (FEM), finite differences method (FDM) or control-volume method (CVM), the fewer numbers of nodes and elements in BEM not only reduces the costs of modeling and discretization, but also decreases the size of the problem which usually results in less computational efforts and memory storage for solution.

Boundary element method has been successfully used to solve Laplace equation in two- and three-dimensions [3]. As there are no body–forces in Laplace equation, when solving it with boundary element method, no domain integration is needed and all advantages of BEM are preserved. Governing equation of seepage in isotropic media is the Laplace equation: $\nabla^2\varphi=0$ in which φ is the potential head [4]. BEM solution of seepage problems in isotropic two-dimensional problems are very well established in the literature, for example see [5,6]. In orthotropic materials that is a special case of anisotropy, the hydraulic conductivities are different in x, y and z directions but the principal axes of the hydraulic conductivities are parallel to the Cartesian system

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Nomenclature

x, y, z	coordinates in original domain
x_*, y_*, z_*	coordinates in rotated domain
x_{**}, y_{**}, z_{**}	coordinates in transformed domain
φ	potential head
p	seeping fluid pressure
ρ	seeping fluid density
$\mathbf{n}$	outward normal to boundary unit vector
$\mathbf{s}, \mathbf{t}$	two tangential to boundary unit vectors
$\partial\varphi/\partial n$	normal to boundary gradient of potential head
$\mathbf{q}$	flux vector
$\mathbf{K}$	hydraulic conductivity tensor
K_{xx}, K_{yy}, K_{zz}	diagonal elements of the hydraulic conductivity tensor
K_{xy}, K_{xz}, K_{yz}	cross-diagonal elements of the hydraulic conductivity tensor
$k_{x_*}, k_{y_*}, k_{z_*}$	hydraulic conductivity in principal axes directions
V_1, V_2, V_3	three eigenvectors of hydraulic conductivity tenor
∇	Nabla operator: $(\partial/\partial x,\ \partial/\partial y,\ \partial/\partial z)$
∇_2	horizontal gradient operator: $(\partial/\partial x,\ \partial/\partial y)$
∇^2	Laplacian operator: $\partial^2/\partial x^2+\partial^2/\partial y^2+\partial^2/\partial z^2$
G	Green's function
det[]	determinant operator of a matrix
I_1, I_2, I_3	invariants of hydraulic conductivity tensor
$\lambda_1, \lambda_2, \lambda_3$	three eigenvalues of hydraulic conductivity tensor
$\mathbf{R}$	rotating transformation matrix
$\mathbf{S}$	stretching transformation matrix
$\mathbf{T}$	direct transformation matrix

of coordinates. Governing equation in orthotropic media is $k_x\partial^2\varphi/\partial x^2+k_y\partial^2\varphi/\partial y^2+k_z\partial^2\varphi/\partial z^2=0$ [7], where k_x, k_y and k_z are the orthotropic hydraulic conductivities in x, y and z directions, respectively. Solution of this equation by boundary element method has been provided by two distinct approaches [8]. The first approach, that we here call it the fundamental solution approach, uses the fundamental solution of the governing equation. By the use of fundamental solution and the Green's theorem, the boundary integral equation can be derived. This boundary integral equation then can be discretized on the boundary for solving the problem by the boundary element method. Using this approach, Brebbia and Chang [7] derived $1/\{4\pi[k_xk_yk_z(x^2/k_x+y^2/k_z+z^2/k_z)]^{1/2}\}$ as the fundamental solution of the governing equation and then solved it directly by BEM. Weakly singular integral equations for Darcy's flow in anisotropic media are given in [9]. Rungamornrat [10] also used the fundamental solution approach and the governing equations were established based on a pair of weakly singular weak-form integral equations for fluid pressure and fluid flux. More complicated set of fundamental solutions for the problem were then used. The second approach, that we here call it the transformation approach, uses a mathematical transformation to transform the governing equation to Laplace equation. After transformation, the problem is solved by classical boundary element method for Laplace equation in the transformed space. Then by an inverse transformation, the calculated results will be transformed back to the original space. Transformation approach has also been used in [11–13] for seepage analysis with boundary elements in two-dimensional orthotropic materials. The transformation approach has been used successfully for seepage applications by Laef et.al. [14] for three-dimensional aquifers. Although the three-dimensional aquifers were dealt with in [14], the effective dimension of an aquifer system was reduced to two by use of the Dupuit assumptions. Laef et.al. [14] discussed that in many practical cases sufficiently accurate results could be obtained with less than a full three-dimensional analysis. Based on the fact that the horizontal dimensions of the aquifer are often much larger than the vertical dimensions, they assumed a 'nearly horizontal' flow as a good approximation. After assuming a nearly horizontal flow, their problem was reduced to two dimensions in the horizontal plane. They used the two-dimensional coordinate transformation to deal with anisotropy. Although the nearly horizontal assumption is a good approximation for some problems like large aquifers, in many other civil engineering structures like dams, sheet-piles, etc. it is not the case because the horizontal dimensions are not much larger than the vertical dimensions. Thus many problems can only be solved by a full three-dimensional analysis. A full three-dimensional seepage analysis of orthotropic materials by BEM has been given by Rafiezadeh and Ataie-Ashtiani [15] using the transformation approach. In their work they used the three-dimensional coordinate transformation for orthotropic materials to transform the governing equation to the Laplace equation.

For the well-posed seepage boundary value problems, the boundary conditions are of Dirichlet type (potential is prescribed) or Neumann type (normal to boundary potential gradient prescribed $(\partial\varphi/\partial n)$). In the fundamental solution approach however the required boundary condition should be one of the two cases: potential should be prescribed or $k_x\partial\varphi/\partial x+k_y\partial\varphi/\partial y+k_z\partial\varphi/\partial z$ should be prescribed [7]. In the second boundary condition type, most of the times the pre-calculation of the boundary condition should be done.

In the transformation approach, Dirichlet boundary condition remains unchanged while the Neumann boundary condition should also be transformed. The transformations of these Neumann boundary conditions are straight-forward and can easily be computed. The transformations are given for two-dimensions by Laef et.al.[14] and Liggett and Liu [5] for general anisotropic media. In three-dimensions, such transformations for orthotropic media are given by Rafiezadeh and Ataie-Ashtiani [15]. In multi-domain problems, corresponding nodes on the interface between neighboring domains should satisfy the compatibility and equilibrium equations. Compatibility equations oblige the equality of the potential on adjacent nodes and equilibrium conditions conserve the flowing normal flux between the two neighboring domains. For any seepage analysis in multi-domain media by BEM, the compatibility and equilibrium equations should be undertaken in the model. When the domains are anisotropic and the transformation approach is used, these equations should also be transformed. The compatibility equation remains unchanged after transformation however, the distortion of boundaries leads to misalignment of the unit outward normal vectors along the interfaces of neighboring domains, the equilibrium of normal fluid fluxes needs to be transformed accordingly. $(k_{xx}^{(1)}k_{yy}^{(1)})^{1/2}(\partial\varphi/\partial n)^{(1)}=(k_{xx}^{(2)}k_{yy}^{(2)})^{1/2}(\partial\varphi/\partial n)^{(2)}$ has been used as the equilibrium equation in [11–13] which is an approximation based on the concept of equivalent hydraulic conductivity in two-dimensional orthotropic domains. Laef et.al. [14] derived the exact transformation of equilibrium equation in two dimensions. Rafiezadeh and Ataie-Ashtiani [15] derived the exact equilibrium condition for three-dimensional orthotropic media.

More general anisotropic media often occur when dealing with applied seepage problems in engineering. Large hydraulic structures like dams are usually built on the rivers flowing in young

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