Engineering Analysis with Boundary Elements 34 (2010) 1049–1057

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Source point isolation boundary element method for solving general anisotropic potential and elastic problems with varying material properties

Xiao-Wei Gao

State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, PR China

ARTICLE INFO

Article history:
Received 1 February 2010
Accepted 30 June 2010
Available online 16 July 2010

Keywords:
Boundary element method
Source point isolation technique
Anisotropic potential problem
Anisotropic elasticity
Varying material property

ABSTRACT

This paper presents a new robust boundary element method, based on a source point isolation technique, for solving general anisotropic potential and elastic problems with varying coefficients. Different types of fundamental solutions can be used to derive the basic integral equations for specific anisotropic problems, although fundamental solutions corresponding to isotropic problems are recommended and adopted in the paper. The use of isotropic fundamental solutions for anisotropic and/or varying material property problems results in domain integrals in the basic integral equations. The radial integration method is employed to transform the domain integrals into boundary integrals, resulting in a pure boundary element analysis algorithm that does not need any internal cells. Numerical examples for 2D and 3D potential and elastic problems are given to demonstrate the correctness and robustness of the proposed method.

1. Introduction

Although the boundary element method (BEM) has been successfully established and applied in engineering as an effective and convenient numerical tool for finding the solution to a wide class of boundary value problems [1], its application to general anisotropic and non-linear problems is not as effective as to linear isotropic problems. The main reason is that the anisotropy of a material increases the number of material properties, and hence makes the fundamental solutions either too complex or unavailable in a closed form [2–4]. To obtain fundamental solutions for general anisotropic solids, Mura [5] investigated line-integral representations of the three-dimensional Green's function in a full-space medium. Based on complex potential theory, Lekhnitskii [6] and Sollero [7] gave anisotropic fundamental solutions based on finding roots of a characteristic fourth degree polynomial equation [8]. Using Lekhnitskii's fundamental solutions, Shiah and Tan solved 2D anisotropic elasticity problems with body forces [9], in which volume integrals involving the body forces are transformed into surface integrals by applying a differentiation technique before doing an analytical transformation. Wang [10] in 1997 derived explicit expressions for three-dimensional elastostatic Green's displacement in general anisotropic solids. In Wang's work, the numerical solution of a polynomial of sixth order is required in order to obtain the entire Green's function. Wang's work is purely theoretical. Later, Tonon et al. [11] applied Wang's work to a boundary element implementation. Also in 1997 [12], Ting and Lee derived explicit expressions for the anisotropic Green's functions in terms of the Stroh eigenvalues [13]. For multi-medium problems, Fares and Li [14] constructed the Green's functions for multilayered media using a general image method. By utilizing an inverse Fourier transform in polar coordinates and combining with Mindlin's superposition method, Pan and Yuan [15] derived an expression of Green's functions for anisotropic half-space and bimaterial problems.

The works described above can result in pure boundary-only integral equations for anisotropic problems with constant material parameters. The drawback of these works is that evaluating a line integral or solving an eigenvalue equation set is cumbersome and often time-consuming. For some problems, deriving a closed form of fundamental solutions, especially stress fundamental solutions, is either very difficult in itself or requires complicated coding of the resulting expression. Besides, for multi-medium problems, a closed form of the fundamental solutions may be unavailable and numerical integration of a line integral may be necessary. This may result in the dependence of Green's functions on material properties [15]. On the other hand, obtaining fundamental solutions for generating pure boundary integral equations may never be possible for anisotropic problems involving varying material properties as occurs in functionally graded material problems (FGMs) [16]. Therefore, a new method needs to be exploited for solving general anisotropic problems associated with advanced materials.

In a different manner, Schclar and Partridge [17] proposed an approach to solve anisotropic problems based on the use of the fundamental solutions, for a homogeneous material, which leads

E-mail address: xwgao@dlut.edu.cn

0955-7997/$ - see front matter
doi:10.1016/j.enganabound.2010.06.015

to a boundary-domain integral equation due to the residuals between the actual and the isotropic quantities. In this approach, to achieve a boundary-only BEM algorithm, the dual reciprocity method (DRM) [18] is used to take the relevant domain integral to the boundary. The main advantage of this approach [17] is that the anisotropic fundamental solution is avoided and, therefore, it is easier to apply in solving different types of anisotropic problems. In a similar way, Chen et al. [19,20] converted the resulting domain integrals to the boundary based on the method of fundamental solutions and DRM. The work in [17] is only suitable for constant material problems, and, since it relies on the use of DRM to convert domain integrals to the boundary, certain amounts of internal points may be required to ensure a satisfactory computational accuracy.

In this paper, a new and simple BEM, named Source-point Isolation Boundary Element Method (SIBEM), is proposed for solving general anisotropic potential and elastic problems. The method is based on a source-point isolation technique and allows for material properties to be variable. Different types of fundamental solutions can be used in the derived integral equations. For constant material property problems, if the fundamental solutions used are the ones of the corresponding problem, pure boundary integral representation can be preserved. However, as used in this paper, if the fundamental solution of the homogeneous material is used for anisotropic problems or for problems, in which material properties are not constant, the resulting integral equation includes domain integrals. In this case, the radial integration method (RIM) [21,22] is adopted to transform the domain integrals into boundary integrals. For large gradient potential/elastic problems, some internal points may be needed to improve the accuracy. However, for general engineering problems, few or even no internal points are needed to obtain a satisfactory result [16]. While both RIM and DRM need to use RBFs to approximate the unknowns included in the domain integrals, it should be pointed out that DRM transforms domain integrals to the boundary by employing particular solutions derived from the differential operator of the problem, whereas an RIM can transform any domain integrals to the boundary based on purely mathematical treatments without needing any particular solutions [23,24]. In numerical examples, 2D and 3D anisotropic potential and elastic problems are given to demonstrate the correctness and robustness of the proposed method. Also, the challenging problem of the honeycomb sandwich structure is particularly introduced to show the ability of the proposed techniques to solve complex 3D geometry problems with strong contrasts in material properties.

2. Boundary-domain integral equations for general potential problems based on source point isolation technique

2.1. Formulations for general potential problems

The governing equation for general potential problems can be expressed as [25]

$$\frac{\partial}{\partial x_i}\left(k_{ij}\frac{\partial T}{\partial x_j}\right)+Q=0 \tag{1}$$

where k_{ij} and Q are the material property tensor and source terms, respectively, and T denotes the potential. They may all be functions of spatial coordinates for non-homogeneous problems or functions of field variables for non-linear problems. It is noted that the material property tensor is symmetric, i.e., $k_{ij}=k_{ji}$. The repeated subscripts in Eq. (1) represent summation over the ranges of their values. Using a weight function G to multiply both sides of Eq. (1) and integrating over the whole domain Ω, the following weak form can be written.

$$\int_\Omega G\frac{\partial}{\partial x_i}\left(k_{ij}\frac{\partial T}{\partial x_j}\right)d\Omega+\int_\Omega GQ\,d\Omega=0 \tag{2}$$

The first integral can be manipulated as follows:

$$\begin{aligned}\int_\Omega G\frac{\partial}{\partial x_i}\left(k_{ij}\frac{\partial T}{\partial x_j}\right)d\Omega&=\int_\Gamma Gk_{ij}\frac{\partial T}{\partial x_j}n_i\,d\Gamma-\int_\Omega\frac{\partial G}{\partial x_i}k_{ij}\frac{\partial T}{\partial x_j}\,d\Omega\\&=-\int_\Gamma Gq\,d\Gamma-\int_\Gamma q^*T\,d\Gamma+\int_\Omega^{-}\frac{\partial}{\partial x_j}\left(k_{ij}\frac{\partial G}{\partial x_i}\right)T\,d\Omega\\&=-\int_\Gamma Gq\,d\Gamma-\int_\Gamma q^*T\,d\Gamma+I_\Omega\end{aligned} \tag{3}$$

where

$$q=-k_{ij}\frac{\partial T}{\partial x_j}n_i$$
$$q^*=\frac{\partial G}{\partial x_i}k_{ij}n_j \tag{4}$$

$$I_\Omega=\int_\Omega^{-}\frac{\partial}{\partial x_j}\left(k_{ij}\frac{\partial G}{\partial x_i}\right)T\,d\Omega \tag{5}$$

in which Γ denotes the boundary of the domain Ω, n_i is the i-th component of the outward normal vector to Γ, and q is the flux. It is noted that the domain integral in Eq. (5) may be strongly singular (depending on the choice of G) and, therefore, a different integral symbol is used to denote this.

We assume that the weight function G is a fundamental solution of either isotropic or anisotropic problems. Usually, it is a function of the distance r between the source point p and field point q [25,26]. When $r\to 0$, G may be singular and, therefore, an infinitesimal circular domain Ω_ε centered at the source point p with radius ε can be isolated from Ω (Fig. 1).

The last term in Eq. (3) now can be written as

$$\begin{aligned}I_\Omega&=\int_\Omega^{-}\frac{\partial}{\partial x_j}\left(k_{ij}\frac{\partial G}{\partial x_i}\right)T\,d\Omega=\lim_{\varepsilon\to 0}\int_{\Omega_\varepsilon}\frac{\partial}{\partial x_j}\left(k_{ij}\frac{\partial G}{\partial x_i}\right)T\,d\Omega\\&\quad+\lim_{\varepsilon\to 0}\int_{\Omega-\Omega_\varepsilon}\frac{\partial}{\partial x_j}\left(k_{ij}\frac{\partial G}{\partial x_i}\right)T\,d\Omega\\&=T(p)\lim_{\varepsilon\to 0}\int_{\Omega_\varepsilon}\frac{\partial}{\partial x_j}\left(k_{ij}\frac{\partial G}{\partial x_i}\right)d\Omega+\int_\Omega\frac{\partial}{\partial x_j}\left(k_{ij}\frac{\partial G}{\partial x_i}\right)T\,d\Omega\\&=-k(p)T(p)+\int_\Omega VT\,d\Omega\end{aligned} \tag{6}$$

where

$$k(p)=-\lim_{\varepsilon\to 0}\int_{\Omega_\varepsilon}\frac{\partial}{\partial x_i}\left(k_{ij}\frac{\partial G}{\partial x_j}\right)d\Omega=-\lim_{\varepsilon\to 0}\int_{\Gamma_\varepsilon}k_{ij}\frac{\partial G}{\partial x_j}n_i\,d\Gamma$$

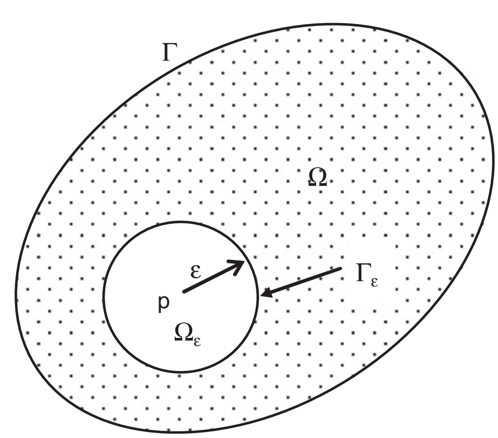

Fig. 1. An infinitesimal domain Ω_ε isolated from Ω.