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Estimation and inference for additive partially nonlinear models

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In this paper, we extend the additive partially linear model to the additive partially nonlinear model in which the linear part of the additive partially linear model is replaced by a nonlinear function of the covariates. A profile nonlinear least squares estimation procedure for the parameter vector in nonlinear function and the nonparametric functions of the additive partially nonlinear model is proposed and the asymptotic properties of the resulting estimators are established. Furthermore, we apply the empirical likelihood method to the additive partially nonlinear model. An empirical likelihood ratio for the parameter vector and a residual adjusted empirical likelihood ratio for the nonparametric functions have been proposed. Wilks phenomenon is proved and the confidence regions for the parametric vector and the nonparametric functions are constructed. Some simulations have been conducted to assess the performance of the proposed estimating procedures. The results have demonstrated that both the procedures perform well in finite samples. Compared with the results from the empirical likelihood method with those from the profile nonlinear least squares method, the empirical likelihood method performs better in terms of coverage probabilities and average widths of confidence bands.

1. Introduction

In the last three decades, semiparametric models have received much attention because they keep the explanatory power of the parametric models and the flexibility of the nonparametric models. One of the most important semiparametric models is the additive partially linear model which takes the form of

$$Y = X^T\beta + \sum_{d=1}^{D} f_d(Z_d) + \varepsilon, \tag{1.1}$$

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where Y is the response variable, $X \in R^p$ and $Z = (Z_1, Z_2, \ldots, Z_D)^T \in R^D$ are covariates, $\beta = (\beta_1, \ldots, \beta_p)^T$ is a p-dimensional vector of unknown parameters, $f_1(\cdot), \ldots, f_D(\cdot)$ are unknown smooth functions, ε is the random error with conditional mean zero given X and Z. The model assumes that the relationship between the response variable Y and its associate covariate X is linear. However, the linear relationship is not complex enough to capture the underlying relationship between the response variable Y and the covariate X in many practical situations. To solve the problem, in this paper we extend the additive partially linear model to the so-called additive partially nonlinear model which can be described as

$$Y = g(X, \beta) + \sum_{d=1}^{D} f_d(Z_d) + \varepsilon, \tag{1.2}$$

where $g(\cdot, \cdot)$ is a pre-specified nonlinear function. Note that X and β in model (1.2) do not necessarily have the same dimension. The model retains the flexibility of the additive model and the interpretability of the nonlinear regression model.

Model (1.2) includes many important statistical models as its special cases. For instance, it reduces to the additive model when $g(\cdot, \cdot) = 0$. Recently, a variety of algorithms such as the backfitting algorithm (Hastie & Tibshirani, 1990; Opsomer & Ruppert, 1999), marginal integration method (Linton & Nielsen, 1995) have been proposed for estimating the nonparametric functions in the additive model. Opsomer and Ruppert (1997) studied the bivariate additive model by the local polynomial regression method. Moreover, if $g(X, \beta) = X^T\beta$, the additive partially nonlinear model becomes the additive partially linear model (1.1). Model (1.1) has been extensively studied in the literature. Besides the above mentioned backfitting algorithm and marginal integration method, Li (2000) proposed the series estimation method for estimating the parameter vector β in model (1.1). Liang, Thurston, and Ruppert (2008) proposed a correction-for-attenuation estimator for β in model (1.1) when X is measured with error. Guo, Tang, and Tian (2013) and Liu, Wang, and Liang (2011) studied the variable selection issues in model (1.1). Furthermore, based on the empirical likelihood method, Liang, Su, and Thurston (2009), Wang, Chen, and Lin (2010) and Wei, Luo, and Wu (2012) considered the construction of the confidence region of β in model (1.1) when the covariate X is measured with additive errors. Zhao and Xue (2013) constructed the confidence intervals of the nonparametric components when the linear covariate X is measured with and without errors. Zhou, Zhao, and Lin (2014) applied the empirical likelihood method to the longitudinal additive partially linear errors-in-variables model. A special case of model (1.2) with $D = 1$, the partially nonlinear model, has gained much attention in recent years. Li and Nie (2007) proposed an estimation procedure for β through a nonlinear mixed-effects approach. Furthermore, Li and Nie (2008) developed two estimation procedures via profile nonlinear least squares and linear approximation approach. Meanwhile, Huang and Chen (2008) considered the spline profile least square estimator of β when the nonparametric function was approximated by some graduating functions. Xiao, Tian, and Li (2014) applied the empirical likelihood method to the inference for the parameter vector and the nonparametric function. When there are no nonparametric functions $f_1(\cdot), \ldots, f_D(\cdot)$, model (1.2) turns into the well known nonlinear regression model. The estimation and inference procedures of the nonlinear regression model can be referred to Bates and Watts (1988) and other relevant references.

In this paper, we are concerned with the estimation and inference issue for the additive partially nonlinear model (1.2). Firstly, motivated by Li and Nie (2008), we propose a profile nonlinear least squares estimation procedure for the parameter vector β and the nonparametric functions in model (1.2), establish the asymptotic properties of the resulting estimators. Specifically, we first rewrite model (1.2) as an additive model by pretending β to be known. By means of the backfitting algorithm, we get the pseudo-backfitting estimators of the nonparametric functions, then the nonlinear least squares approach is used to estimate the parameter vector β by replacing the nonparametric functions in model (1.2) with their pseudo-backfitting estimators. Secondly, we apply the empirical likelihood method to construct the confidence region for the parameter vector β and the nonparametric functions. It is well known that the empirical likelihood method which was introduced by Owen (1988, 1990) enjoys many nice features in the construction of confidence region, for example, it does not impose prior constraints on the shape of confidence region, it does not require the construction of a pivotal quantity and it does not involve a plug-in estimator for the asymptotic covariance matrix. For these reasons, the empirical likelihood method has found many applications in many kinds of models such as linear models (Owen, 1991; Wang & Rao, 2002), partially linear models (Li & Xue, 2008; Shi & Lau, 2000; Xue & Xue, 2011), varying-coefficient partially linear models (Huang & Zhang, 2009; Wang, Li, & Lin, 2011; Wei & Mei, 2012; Zhao & Xue, 2009), additive partially linear models (Wang et al., 2010; Wei et al., 2012; Zhou et al., 2014) and so on. Finally, we conduct some simulations to assess the performance of the proposed nonlinear least squares estimation procedure and the empirical likelihood method.

The rest of this paper is organized as follows. In Section 2, we introduce the profile nonlinear least squares procedure for the parameter vector β and the nonparametric functions, and study the relevant asymptotic properties. In Section 3, we apply the empirical likelihood method to the parameter vector β and the nonparametric functions. The empirical log-likelihood ratio for β is defined and its asymptotic distribution is derived. The corresponding confidence region for β is constructed. Furthermore, the residual adjusted empirical log-likelihood ratio for the nonparametric functions is developed and the corresponding confidence region is constructed. Section 4 provides examples based on simulated data, a comparison between the proposed nonlinear least squares estimation approach and the empirical likelihood method is performed in terms of coverage accuracy and areas widths of confidence regions bands. Concluding comments are given in Section 5 and the proofs of the main results are presented in the Appendix.