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A CUSUM test for panel mean change detection

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ABSTRACT

A test for panel structural mean change is developed from the CUSUM of the panel processes. Limiting null distribution and consistency of the test are established. The test is shown to have stable finite sample sizes than the existing test of Horvath and Huskova (2012) based on the squared CUSUM. If the mean changes are not cancelled in that their average is away from zero, the proposed test has better power than the existing test. On the other hand, if the mean changes are nearly cancelled, the existing test has better power. The proposed tests are illustrated by a real data set analysis.

1. Introduction

Structural change problems in panel data models are important issues for economic or financial data analysis because a big macroeconomic policy change or a financial crisis has simultaneous influence on many economic or financial variables. Researches for structural change problems in panel models have been activated due to a vast amount of data in the modern economic world or financial markets. Some studies were made by Bai (1997), Bai et al. (1998) and Han and Park (1989) for change point estimation and testing of multivariate time series models and by Emerson and Kao (2001, 2002) for testing of structural change of a time trend regression in panel data.

Recently, structural change detection problems in mean and variance of panel data have been investigated by some authors. Bai (2010) studied estimation for common change point in mean and variance in panel data. Horvath and Huskova (2012), following Bai (2010)'s quasi-maximum likelihood argument, developed a test for panel mean change, based on the *squared* cumulative sum (squared CUSUM) of the panel processes. Li, Tian, Xiao, and Chen (2015) and Shi (2015) proposed tests for panel variance changes, based on the CUSUM and the squared CUSUM, respectively, of the squared panel processes.

We note that the test of Horvath and Huskova (2012), being based on the squared CUSUM, has good power against average squared change away from 0. However, in practice, one may be more interested in detecting average change than average squared change. For that purpose, we will construct a simple panel mean change detection test based on the CUSUM of the panel processes. As well as the limiting null distribution, consistency of the proposed test will be established against average change away from 0. The proposed test, being based on the CUSUM, has good power against average changes away from 0.

Aimed at different targets of average squared changes and average changes, none of the existing test of Horvath and Huskova (2012) and the proposed test does not dominate the other one in power performance. The existing test has power

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advantage over the proposed test against changes which cancel to near-zero sum. The proposed test has power advantage over the existing test against non-cancelling changes.

We claim that non-cancelling changes are more frequent than cancelling change in practice. The mean changes in the panel system are usually caused by a big shock which shifts all means to a common direction: a good big shock shifts all panel units to a good direction and a bad big shock acts reversely. For example, the Korean IMF economic crisis in the years 1997–1998 caused bad effects on almost all Korean stock prices, foreign exchange rates, house values, etc. Similar bad effects of the world wide financial crisis in the years 2007–2008 can be observed on stock prices and house values.

A Monte Carlo experiment compares the two tests. It shows better power performance of the proposed test than the existing test against non-cancelling changes and reversed power performance against cancelling changes. Moreover, it reveals that the proposed test has significantly better size performance in case of serially correlated panels.

The panel volatility change tests of Li et al. (2015) and Shi (2015) are compared in the context of mean changes of the squared process showing the same relative performance as the panel mean change tests: in case of non-cancelling volatility changes, the test of Li et al. (2015) based on the CUSUM of squared process has better power than the test of Shi (2015) based on the squared CUSUM of squared process; in case of cancelling volatility changes, the test of Shi (2015) has better power than the test of Li et al. (2015).

The remaining of the paper is organized as follows. Section 2 discusses the mean change detection with main theoretical results. Section 3 deals with Monte Carlo comparisons. Section 4 compares the tests for volatility change detections. Section 5 illustrates the proposed tests with a real data set. Section 6 concludes.

2. Mean change detection

We consider a panel data model consisting of n panels and T observations on each panel unit as given by

$$X_{it} = \mu_i + \sigma_i(\delta_i \mathbb{I}\{t > t_0\} + u_{it}), \quad 1 \le i \le n, \ 1 \le t \le T \tag{1}$$

where $t_0 \in \{1, \ldots, T\}$ is unknown common change point, $\mathbb{I}\{t > t_0\}$ is the indicator function of $\{t > t_0\}$, and, for each i, $\{u_{it}\}$ is a stationary process with $Eu_{it} = 0$ and $Var(u_{it}) = 1$. As in Horvath and Huskova (2012), Li et al. (2015), and Shi (2015), we assume that $\{u_{it}, -\infty < t < \infty\}$ are cross-sectionally independent but can be serially correlated. We have $E(X_{it}) = \mu_i$ for $1 \le t \le t_0$; $E(X_{it}) = \mu_i + \sigma_i\delta_i$ for $t_0 < t \le T$; and $Var(X_{it}) = \sigma_i^2$. The mean change parameter δ_i is standardized mean change in $E(X_{it})$. We wish to test the null hypothesis

$$H_0 : \delta_i = 0 \quad \text{for all } 1 \le i \le n$$

that the mean $E(X_{it})$ will not change during the observation period.

Horvath and Huskova (2012) developed a test by applying the quasi-maximum likelihood argument of Bai (2010) which is based on the **squared cumulative sum** process

$$\bar{H}_{nT}(z) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \left\{ \frac{1}{v_i^2} Z_{iT}^2(z) - \frac{[Tz](T-[Tz])}{T^2} \right\}, \quad 0 \le z \le 1, \tag{2}$$

where

$$Z_{iT}(z) = \frac{1}{\sqrt{T}} \sum_{t=1}^{[Tz]} (X_{it} - \bar{X}_i), \quad \text{with } \bar{X}_i = \frac{1}{T}\sum_{t=1}^{T} X_{it},$$

is the cumulative sum process and $v_i^2 = \lim_{T\to\infty} Var\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T} X_{it}\right)$ is the long-run variance of X_{it} for $i = 1, \ldots, n$. Note that $Z_{iT}(z)$ is standardized by the long-run standard deviation v_i in order to adjust serial correlation in X_{it}.

As proved by Horvath and Huskova (2012, Theorem 3) for consistency of the sup test based on the squared CUSUM process, say Sup H, against changes such that $n^{-1/2}T\sum_{i=1}^{n}\delta_i^2 \to \infty$, the test has good power for detecting average squared changes $\bar{\delta}^Q = n^{-1}\sum_{i=1}^{n}\delta_i^2$. However, even though the test Sup H has good power against average squared changes, the Sup H test remains to be improved for detecting other changes in some class of important alternatives of nonnegative (or nonpositive) panel mean changes such as those caused by the world wide financial crisis or the Korean IMF economic crisis mentioned in Section 1.

Such nonnegative or nonpositive panel mean changes may be more well detected by a test designed to detect average change $\bar{\delta} = n^{-1}\sum_{i=1}^{n}\delta_i$ than by the Sup H test designed to detect average squared change $\bar{\delta}^Q$. Average change is well detected by a test designed to detect a common change $\delta_1 = \cdots = \delta_n = \delta$. From (1), we have $(X_{it}-\mu_i)/\sigma_i = \delta_i\mathbb{I}\{t > t_0\}+u_{it}$. Note that, if μ_i are all known, then the test problem for the common change δ becomes that of the summed univariate time series model $\sum_{i=1}^{n}(X_{it}-\mu_i)/\sigma_i = \sum_{i=1}^{n}\delta\mathbb{I}\{t > t_0\} + \sum_{i=1}^{n}u_{it}$ for which the Sup test and the CUSUM test are all based on the cumulative sum process $B_{nT}(z) = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left\{\frac{1}{v_i}\frac{1}{\sqrt{T}}\sum_{t=1}^{[Tz]}(X_{it}-\mu_i)\right\}$. For the real situation of unknown μ_i, the unknown μ_i are replaced by $\bar{X}_i$ and a natural test for the common change is constructed from the **cumulative sum** process

$$\bar{B}_{nT}(z) = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left\{\frac{1}{v_i}Z_{iT}(z)\right\}. \tag{3}$$