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An integral-equation formulation of nonlinear deformation in a stack of buffered plates

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ABSTRACT

An integral-equation formulation has been derived for nonlinear deformation in a stack of buffered Kirchhoff plates. The plates are assumed to follow a nonlinear bending moment-curvature law and the buffer material to follow the generalized Hooke's law. By employing the recently derived special Green's function for multilayers with interfacial membrane and flexural rigidities as the kernel, the integral-equation formulation only involves the surface loading area (for application to an indentation problem) and the portion of plates undergoing nonlinear deformation. Based on the integral equation, an efficient and accurate boundary element method has been derived to numerically solve the cylindrical indentation problem of the material with a bilinear flexural bending law for the plates. Numerical examples are presented to show a progressive damage process of yielding across a stack of plates as well as to demonstrate the validity and accuracy of the present integral-equation formulation.

1. Introduction

Layers sandwiched by parallel plates, or say, a stack of buffered parallel plates, can be found in many engineered materials and structures for applications to vibration/noise damping, weight reduction, impact protection, etc. [1–3]. Most recently, Yang et al. [4] proposed to model graphite [5,6] at the nanometer scale as a stack of buffered plates. The plates represent the covalent bonding effect within individual graphene sheets, whilst the buffer material models the Van der Waals interaction between adjacent graphene sheets. This model can also be extended to other nanolayered materials similar to graphite [7–11]. These materials are strongly anisotropic, if not elastically, then upon plastic deformation and delamination across the basal planes. They analyzed the cylindrical indentation problem of graphite at the nanoscale, but only in the linear elastic limit. Strong concentration of loading was predicted to occur in the plates at the edges of contact. Naturally nonlinear deformation and damage are expected to initiate at those locations. In the present study, we derive an integral-equation formulation of this composite system with the plates capable of undergoing nonlinear deformation in the flexural mode. Furthermore, we derive an efficient and accurate boundary element method based on the integral equations to numerically solve the nonlinear deformation problem in a stack of buffered plates under indentation. Since the recently derived special Green's function for multilayers with interfacial membrane and flexural rigidities [4,12] is taken as the integral kernel, the numerical scheme is efficient by only involving the indentation loading area and the part of plates undergoing nonlinear deformation. However, while focusing on the integral-equation formulation, this work is not intended to analyze the behavior of graphite or a similar material with realistic nonlinear constitutive laws, which has not been available in the literature.

The rest of the paper is organized as follows. In Section 2, the model material consisting of a stack of buffered plates is described. Plastic bending is considered for the plates. In Section 3, an integral equation is derived that expresses the displacement field in the buffer layers as an integral over the indentation contact area and the portion of plates undergoing nonlinear deformation. In Section 4, an efficient and accurate boundary element method is derived based on the integral-equation formulation. It is capable of solving for the damage processes as well as the deformation field in the material under indentation. In Section 5, numerical examples are presented to show a progressive damage process across a stack of plates and to demonstrate the validity and accuracy of the formulation. In Section 6, conclusions are finally drawn.

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2. Model material consisting of a stack of buffered plates

A semi-infinite composite system consisting of a stack of parallel Kirchhoff plates buffered by a linearly elastic and isotropic material is considered, as schematically shown in Fig. 1. The plane-strain condition is assumed. The Kirchhoff plates are linear in the in-plane membrane deformation mode but nonlinear in the flexural bending mode. The Cartesian coordinate system is established with the x_1-axis parallel to and the x_3-axis normal and pointing into the surface. The x_2-axis is the direction of trivial deformation.

The two-phase composite structure requires two sets of equilibrium equations for the plates and the buffer material. The set of equilibrium equations for the plates that may deform longitudinally and bend about the x_2-axis are given by [13]

$$N_{,1}+f=0; \quad M_{,11}+q=0, \tag{1}$$

where N is the x_1-component of the membrane stress (of the dimension of force/length), M the bending moment parallel to the x_2-axis, and f and q are the longitudinal and transverse components of the body force density on a plate, respectively. The comma in the subscript denotes the partial differentiation with respect to the indices that follow.

The linear in-plane constitutive law of the plates is given by

$$N=E' u^{(p)}_{1,1}, \tag{2}$$

where E' is the membrane stiffness (under the plane-strain condition), and $u^{(p)}$ is the longitudinal displacement component of the plate.

The nonlinear flexural constitutive law of the plates is given by

$$M=D(k-k_0), \tag{3}$$

where D is the flexural rigidity, $k\,(\equiv -w_{,11})$ the curvature, and k_0 a damage parameter. In the definition of k, w is the transverse displacement component u_3, i.e., deflection, of the plate. D is set to be a constant. k_0 may change with loading as described later. It may be understood as the residual curvature in a plate upon yielding. Although the following integral equations can be derived for an arbitrary damage evolution law of the plates, a bilinear flexural bending law is assumed and will be implemented in later simulations. The bilinear flexural bending law is sketched in Fig. 2.

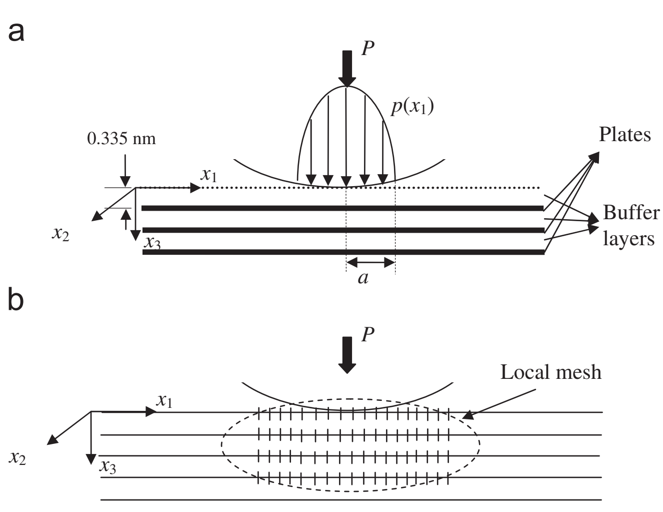

Fig. 1. (a) A stack of buffered Kirchhoff plates under cylindrical indentation; (b) discretization of potential areas of indentation contact and nonlinear plate deformation.

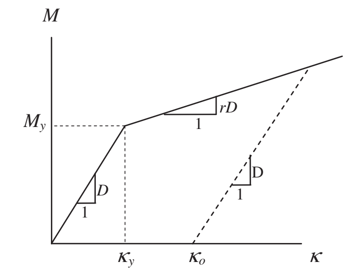

Fig. 2. A bilinear bending moment-curvature law for flexural deformation of a plate.

The plates are initially linearly elastic. They yield at a critical moment M_y (and correspondingly a critical curvature k_y) beyond which work-hardening occurs at a constant ratio r. By specifying the work-hardening ratio r one can determine the damage parameter k_0 from the current curvature k on a monotonic loading path as

$$k_0=\begin{cases} 0 & \text{for} |k|<k_y \\ (1-r)(k-k_y) & \text{otherwise} \end{cases}. \tag{4}$$

The equilibrium equations for the buffer material in the absence of body force are given by

$$\sigma_{ij,j}=0, \tag{5}$$

where σ_{ij} is the stress component, and the repeated indices indicate the Einstein convention of summation. The constitutive law of the buffer material is given by

$$\sigma_{ij}=C_{ijkl}u_{k,l}, \tag{6}$$

where C_{ijkl} is the elastic constant, and u_k the displacement component of the buffer layer. Since the plane-strain condition is assumed, the Latin indices all take values 1 and 3—value 2 is trivial—within the coordinate system as defined in Fig. 1.

The continuity conditions of displacement and traction are enforced at the interfaces between the plates and the buffer layers. It results in the continuity condition of displacement and the discontinuity condition of traction across a plate as

$$\Delta u_i=0, \tag{7}$$

$$\Delta p_1=f \text{ and } \Delta p_3=q. \tag{8}$$

By substituting Eqs. (1)–(3) into Eq. (8) and realizing that the displacements of the plate and the layer at the interfaces are the same, Eq. (8) is recast into

$$\Delta p_i=A_{ij}u_j+p_i^0, \tag{9}$$

with

$$[A_{ij}]=\begin{bmatrix} -E\dfrac{\partial^2}{\partial x_1^2} & 0 \\ 0 & D\dfrac{\partial^4}{\partial x_1^4} \end{bmatrix} \text{ with } i,j=1,3 \tag{10}$$

$$\{p_i^0\}=\begin{Bmatrix} 0 \\ Dk_{0,11} \end{Bmatrix} \text{ with } i=1,3. \tag{11}$$