Journal of Multivariate Analysis 155 (2017) 1–11

Contents lists available at ScienceDirect

Journal of Multivariate Analysis

journal homepage: www.elsevier.com/locate/jmva

Weak convergence of multivariate partial maxima processes

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ARTICLE INFO

Article history:
Received 13 July 2016
Available online 27 November 2016

AMS 2010 subject classifications:
60F17
60G52
60G70

Keywords:
Functional limit theorem
Regular variation
Weak M_1 topology
Extremal process
Weak convergence
Multivariate GARCH

ABSTRACT

For a strictly stationary sequence of $\mathbb{R}_+^d$–valued random vectors we derive functional convergence of partial maxima stochastic processes under joint regular variation and weak dependence conditions. The limit process is an extremal process and the convergence takes place in the space of $\mathbb{R}_+^d$–valued càdlàg functions on $[0, 1]$, with the Skorohod weak M_1 topology. We also show that this topology in general cannot be replaced by the stronger (standard) M_1 topology. The theory is illustrated on three examples, including the multivariate squared GARCH process with constant conditional correlations.

1. Introduction

A classical question in extreme-value theory is under what assumptions the scaled maximum

$$\bigvee_{i=1}^{n} \frac{X_i - b_n}{a_n}$$

of i.i.d. random variables $(X_i)_{i\in\mathbb{N}}$ converges weakly, for some $a_n > 0$ and $b_n \in \mathbb{R}$. Also what are the possible limit distributions? Answers to these questions were given by Fisher and Tippett [14], Gnedenko [15] and de Haan [11]. Introducing a time variable, Lamperti [19] studied the asymptotic distributional behavior of partial maxima stochastic processes

$$\bigvee_{i=1}^{\lfloor nt\rfloor} \frac{X_i - b_n}{a_n}, \quad t \geq 0.$$

Extensions of the theory to dependent random variables, and then to multivariate and spatial settings were particularly stimulating and useful in applications; we refer here only to Adler [1], Leadbetter [20,21], Beirlant et al. [7], de Haan and Ferreira [12] and Resnick [23].

In this paper we focus on the multivariate case in the weakly dependent setting. Let $\mathbb{R}_+^d = [0, \infty)^d$. We consider a stationary sequence of $\mathbb{R}_+^d$-valued random vectors (X_n). In the i.i.d. case it is well known that weak convergence of the scaled maximum is equivalent to the regular variation of the distribution of X_1, i.e.,

$$M_n = \bigvee_{i=1}^{n} \frac{X_i}{a_n} \xrightarrow{d} Y_0$$

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http://dx.doi.org/10.1016/j.jmva.2016.11.012
0047-259X/

if and only if

$$n\,\Pr\Big(\frac{X_1}{a_n} \in \cdot\,\Big) \xrightarrow{v} \mu(\,\cdot\,), \qquad (1)$$

where Y_0 is a random vector with distribution function $F_0(x) = e^{-\mu([0,x]^c)}$, $x \in \mathbb{R}_+^d$, μ is a Radon measure and (a_n) a sequence of positive real numbers such that

$$n\,\Pr(\|X_1\| > a_n) \to 1 \quad \text{as } n \to \infty;$$

see Proposition 7.1 in Resnick [23]. The arrow "$\xrightarrow{v}$" above denotes vague convergence of measures, and $[a, b]$ the product segment, i.e.,

$$[a, b] = [a_1, b_1] \times \cdots \times [a_d, b_d]$$

for $a = (a_1, \ldots, a_d)$, $b = (b_1, \ldots, b_d) \in \mathbb{R}_+^d$.

In the i.i.d. case relation (1) is also equivalent to the functional convergence of stochastic processes of partial maxima of (X_n), i.e.,

$$M_n(\,\cdot\,) = \bigvee_{i=1}^{\lfloor n\cdot\rfloor} \frac{X_i}{a_n} \xrightarrow{d} Y_0(\,\cdot\,) \qquad (2)$$

in $D([0, 1], \mathbb{R}_+^d)$, the space of $\mathbb{R}_+^d$-valued càdlàg functions on $[0, 1]$, with the Skorohod J_1 topology, with the limit Y_0 being an extremal process; see Proposition 7.2 in [23].

In this paper we are interested in the investigation of the asymptotic distributional behavior of the processes M_n for a sequence of weakly dependent $\mathbb{R}_+^d$-valued random vectors that are jointly regularly varying. Since we study extremes of random processes, nonnegativity of the components of random vectors X_n in reality is not a restrictive assumption.

First, we introduce the essential ingredients about regular variation, weak dependence and Skorohod topologies in Section 2. In Section 3 we prove the so-called timeless result on weak convergence of scaled extremes M_n, based on a point process convergence obtained by Davis and Mikosch [10]. Using a multivariate version of the limit theorem derived by Basrak et al. [5] for a certain time–space point processes, in Section 4 we prove a functional limit theorem for processes of partial maxima M_n in the space $D([0, 1], \mathbb{R}_+^d)$ endowed with the Skorohod weak M_1 topology. The methods used are partly based on the work of Basrak and Krizmanić [4] for partial sums. Finally, in Section 5 the theory is applied to m-dependent processes, stochastic recurrence equations and multivariate squared GARCH (p, q) with constant conditional correlations. We also illustrate with an example that the weak M_1 convergence in our main theorem, in general, cannot be replaced by the standard M_1 convergence.

2. Preliminaries

In this section we introduce some basic notions and results on regular variation, point processes and Skorohod topologies that will be used in the following sections. For two vectors $y = (y_1, \ldots, y_d)$ and $z = (z_1, \ldots, z_d)$, $y \le z$ means $y_k \le z_k$ for all $k = 1, \ldots, d$.

2.1. Regular variation

Regular variation on $\mathbb{R}_+^d$ for random vectors is typically formulated in terms of vague convergence on $\mathbb{E}^d = [0, \infty]^d \setminus \{\mathbf{0}\}$. The topology on $\mathbb{E}^d$ is chosen so that a set $B \subseteq \mathbb{E}^d$ has compact closure if and only if it is bounded away from zero, that is, if there exists $u > 0$ such that $B \subseteq \mathbb{E}_u^d = \{x \in \mathbb{E}^d : \|x\| > u\}$. Here $\|\cdot\|$ denotes the max-norm on $\mathbb{R}_+^d$, i.e., $\|x\| = \max\{x_i : i = 1, \ldots, d\}$ where $x = (x_1, \ldots, x_d) \in \mathbb{R}_+^d$. Denote by $C_K^+(\mathbb{E}^d)$ the class of all $\mathbb{R}_+$-valued continuous functions on $\mathbb{E}^d$ with compact support.

The vector ξ with values in $\mathbb{R}_+^d$ is (multivariate) regularly varying with index $\alpha > 0$ if there exists a random vector Θ on the unit sphere $\mathbb{S}_+^{d-1} = \{x \in \mathbb{R}_+^d : \|x\| = 1\}$ in $\mathbb{R}_+^d$, such that for every $u \in (0, \infty)$

$$\frac{\Pr(\|\xi\| > ux,\ \xi/\|\xi\| \in \cdot\,)}{\Pr(\|\xi\| > x)} \rightsquigarrow u^{-\alpha}\,\Pr(\Theta \in \cdot\,) \qquad (3)$$

as $x \to \infty$, where the arrow "$\rightsquigarrow$" denotes weak convergence of finite measures. Regular variation can be expressed in terms of vague convergence of measures on $\mathcal{B}(\mathbb{E}^d)$:

$$n\,\Pr(a_n^{-1}\xi \in \cdot\,) \xrightarrow{v} \mu(\,\cdot\,),$$

where (a_n) is a sequence of positive real numbers tending to infinity and μ is a non-null Radon measure on $\mathcal{B}(\mathbb{E}^d)$.

We say that a strictly stationary $\mathbb{R}_+^d$-valued process $(\xi_n)_{n\in\mathbb{Z}}$ is *jointly regularly varying* with index $\alpha > 0$ if for any nonnegative integer k the kd-dimensional random vector $\xi = (\xi_1, \ldots, \xi_k)$ is multivariate regularly varying with index α.