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Reduced form vector directional quantiles

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ABSTRACT

In this paper, we develop a reduced form multivariate quantile model, using a directional quantile framework. The proposed model is the solution to a collection of directional quantile models for a fixed orthonormal basis, in which each component represents a directional quantile that corresponds to a particular endogenous variable. The model thus delivers a map from the space of exogenous variables (or the σ-field generated by the information available at a particular time) and a unit ball whose dimension is given by the number of endogenous variables, to the space of endogenous variables. The main effect of interest is that of exogenous variables on the vector of endogenous variables, which depends on a multivariate quantile index. An estimator is proposed, using quantile regression time series models, and we study its asymptotic properties. The estimator is then applied to study the interdependence among countries in the European sovereign bonds credit default swap market.

1. Introduction

Lags occur in time series for several reasons, including price stickiness, psychological inertia, permanent vs. transitory shocks, adjustment costs, and delays in implementing new technologies. Modeling dynamic behavior has been a concern in econometrics, and constant-coefficient linear time-series models play a large role. Further, an important way to summarize the dynamics of macroeconomic data is to make use of a vector autoregressive (VAR) model. The VAR approach provides statistical tools for data description, forecasting, and structural inference to study rich dynamics in multivariate time-series models.

Nevertheless, the use of a constant-coefficient model as representative of time-series models may not be adequate, as these models ignore the effects that a succession of small and varied shocks may have on the structure of dynamic economic models, particularly for highly aggregated data series. Moreover, these models cannot appropriately account for the presence of asymmetric dynamic responses. Of particular interest is the asymmetric business cycle dynamics of economic variables, as the occurrence of asymmetries may call into question the usefulness of models with time invariant structures as means of modeling such series.

Quantile regression (QR) is a statistical method for estimating models of conditional quantile functions. This method offers a systematic strategy for examining how covariates influence the location, scale, and shape of the entire response distribution, thereby exposing a variety of heterogeneity in response dynamics. For a given cumulative distribution function F_Y of a univariate random variable Y, the univariate quantile function is well defined. In particular, the τ-quantile for

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$\tau \in (0, 1)$ is defined as $Q_Y(\tau) = \inf\{y \in \mathcal{Y} : \tau \le F_Y(y)\}$, and if F_Y is continuous, then $Q_Y(\tau) = F_Y^{-1}(\tau)$. In the multivariate case, however, there is no unique definition of a multivariate quantile function.

There is a growing literature on the estimation of QR models for multivariate random variables. Hallin et al. [14] and Paindaveine and Šiman [19,20] build on the definition of directional quantiles, whereby quantiles are equipped with a directional vector. Distributional features are thus explored by considering different directional models; see also [9] for related work. Wei [23] develops a bivariate quantile model, following the marginal-conditional structure of Ma and Koenker [18]. White et al. [24] develop an autoregressive model of the quantiles themselves, extending the CAViaR model of Engle and Manganelli [7] to the multivariate case. In related work, Han et al. [15] study quantile dependence among time-series models. Carlier et al. [3] propose a vector QR (linear) model that produces a monotone map, the gradient of a convex function. In a more general setup, Chernozhukov et al. [5] develop a concept of multivariate quantile based on transportation maps between a distribution of interest with a domain in multivariate real numbers and a unit ball of the same dimension. Finally, another approach is to use copula-based quantile models, as any multivariate distribution can be decomposed into its marginals and a dependence function or copula; see, e.g., [1,12,13]; however, such an approach requires imposing distributional assumptions.

The purpose of this paper is to generalize to the multivariate case the quantile autoregressive framework proposed by Koenker and Xiao [16] and Galvao et al. [11]. We develop a reduced form vector directional quantile (VDQ) model based on the multivariate directional quantiles of [14]. The definition of the VDQ model is based on a system of univariate directional quantiles, and, as such, it satisfies some of the monotonicity properties desired in a multivariate setting. We argue that this definition is natural in time-series contexts for which we are interested in estimating a reduced form model.

The proposed VDQ model is a solution to a collection of directional quantile models for a fixed orthonormal basis, in which each component represents a directional quantile that corresponds to a particular endogenous variable. The model thus delivers a map from the space of exogenous variables (or the σ-field generated by the information available at a particular time) and the unit ball whose dimension is given by the number of endogenous variables to the space of endogenous variables. The main effect of interest is that of exogenous variables on the endogenous variables vector, which depends on multivariate quantile indexes.

We apply the VDQ estimator to model European sovereign bonds interdependence. In particular, we propose a multivariate model for the sovereign bonds credit default swaps of Greece and Spain. We also study the effect of Euro-area monetary variables on those countries' sovereign bonds as a means to explore heterogeneity of monetary shocks.

The paper is organized as follows. Section 2 presents the theory of directional quantiles and the definition of the VDQ model. Section 3 provides the development of the case of a bivariate model. An investigation of its monotonicity properties is then given in Section 4. Section 5 describes the asymptotic theory. Section 6 presents an application of the VDQ to the European sovereign risk credit default swap market. Section 7 concludes.

2. Model

Consider an m-dimensional process $\boldsymbol{Y}_t = (Y_{1t}, \ldots, Y_{mt})^\top$ and assume that for all $t \in \{0, 1, \ldots\}$, $\boldsymbol{Y}_t \in \mathcal{Y} \subseteq \mathbb{R}^m$. Further, consider a $k \times 1$ vector of covariates $\boldsymbol{X}_t \in \mathcal{X} \subseteq \mathbb{R}^k$. Our goal is to develop a model for the conditional random variable $\boldsymbol{Y}_t|\boldsymbol{X}_t$. In particular, we seek to define and estimate the multivariate conditional quantile function of $\boldsymbol{Y}_t|\boldsymbol{X}_t$.

Of particular interest is the case of the covariates generated by the σ-field given by $(\boldsymbol{Y}_s : s < t)$ and all other information available at time t. One then deals with a vector autoregressive quantile model. For an autoregressive model of order p, $\boldsymbol{X}_t = (\boldsymbol{Y}_{t-1}^\top, \ldots, \boldsymbol{Y}_{t-p}^\top)^\top$ and $k = mp$, or, if we consider d exogenous covariates $\boldsymbol{Z}_t$, then $\boldsymbol{X}_t = (\boldsymbol{Y}_{t-1}^\top, \ldots, \boldsymbol{Y}_{t-p}^\top, \boldsymbol{Z}_t^\top) \in \mathcal{Z} \subseteq \mathbb{R}^d$ and $k = mp + d$.

Let the vector $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_m)$ be an index of the $\mathbb{R}^m$ space, which is an element of the open unit ball in $\mathbb{R}^m$ (deprived of the origin) $\mathcal{T}^m = \{\boldsymbol{z} \in \mathbb{R}^m : 0 < \|\boldsymbol{z}\| < 1\}$, where $\|\cdot\|$ denotes the Euclidean norm. Our interest lies in defining and estimating

$$\boldsymbol{Q}_{\boldsymbol{Y}_t|\boldsymbol{X}_t}(\boldsymbol{\tau}|\boldsymbol{X}_t) = \boldsymbol{B}(\boldsymbol{\tau})\boldsymbol{X}_t + \boldsymbol{A}(\boldsymbol{\tau}), \tag{1}$$

where $\boldsymbol{Q}$ is an $m \times 1$ vector, which corresponds to the multivariate quantiles of the m random variables, $\boldsymbol{B}(\boldsymbol{\tau}) = (\boldsymbol{B}_1(\boldsymbol{\tau}), \ldots, \boldsymbol{B}_m(\boldsymbol{\tau}))^\top$ is an $m \times k$ matrix of coefficients with $\boldsymbol{B}_j(\boldsymbol{\tau})$ for each $j \in \{1, \ldots, m\}$, the corresponding $k \times 1$ vector of coefficients of the jth element in $\boldsymbol{Y}$, and $\boldsymbol{A}(\boldsymbol{\tau})$ is an $m \times 1$ vector of coefficients. Thus, $\boldsymbol{Q}$ is a map $\mathcal{X} \times \mathcal{T}^m \mapsto \mathcal{Y}$ and corresponds to our proposed definition of multivariate quantiles, the VDQs.

Our definition builds on the work of Hallin et al. [14], who propose to analyze the distributional features of multivariate response variables using the directional quantiles notion of [4,17,23] and others. Quantiles are analyzed in terms of a quantile magnitude and a direction. The vector $\boldsymbol{\tau}$ factorizes into $\boldsymbol{\tau} = \tau \boldsymbol{v}$, where $\tau = \|\boldsymbol{\tau}\| \in (0, 1)$ and $\boldsymbol{v} \in \{\boldsymbol{z} \in \mathbb{R}^m : \|\boldsymbol{z}\| = 1\}$. Then, τ represents a scalar quantile index, and $\boldsymbol{v}$ is an $m \times 1$ directional vector. We define $\boldsymbol{\Gamma}_{\boldsymbol{v}}$ as an $m \times (m-1)$-dimensional matrix, such that $(\boldsymbol{v}, \boldsymbol{\Gamma}_{\boldsymbol{v}})$ forms an orthonormal basis. Note that $\boldsymbol{\Gamma}_{\boldsymbol{v}}$ is not unique but, rather, any $m \times (m-1)$ matrix whose columns are orthogonal to $\boldsymbol{v}$ and to each other.

Following [14], we define the directional regression quantiles as the directional hyperplanes

$$\pi_{(\tau,\boldsymbol{v})} = \left\{(\boldsymbol{x}^\top, \boldsymbol{y}^\top)^\top \in \mathbb{R}^{k+m} : \boldsymbol{v}^\top \boldsymbol{y} = c(\tau, \boldsymbol{v}, \boldsymbol{\Gamma}_{\boldsymbol{v}})^\top \boldsymbol{\Gamma}_{\boldsymbol{v}}^\top \boldsymbol{y} + \boldsymbol{b}(\tau, \boldsymbol{v}, \boldsymbol{\Gamma}_{\boldsymbol{v}})^\top \boldsymbol{x} + a(\tau, \boldsymbol{v})\right\}$$