Engineering Analysis with Boundary Elements 37 (2013) 319–326

Contents lists available at SciVerse ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Large-scale thermal analysis of fiber composites using a line-inclusion model by the fast boundary element method

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A R T I C L E I N F O

Article history:
Received 4 September 2012
Accepted 14 November 2012
Available online 12 December 2012

Keywords:
Boundary element
Fiber composites
Line inclusion
Fast multipole method

A B S T R A C T

The fast boundary element method is applied for the three-dimensional large-scale thermal analysis of fiber-reinforced composites based on a line inclusion model. In this approach, fibers are treated as inclusions with temperature assumed constant over the circular cross-section and varying along the length direction. Therefore, fibers can be meshed with line elements, making both the modeling complexity and the number of unknowns significantly reduced. An interface integral boundary element method introduced by Gao in 2009 (Engineering Analysis with Boundary Elements 2009; 33: 539–546) is extended to generate a single-domain boundary integral equation for governing this line-inclusion problem. Thus in principle, fibers with arbitrary length can be modeled. The fast multipole method is employed for the fast analysis of such problems with large-scales. The largest composite model in a personal desktop computer has the number of fibers reaching 20,000. Numerical results clearly demonstrate validity of the proposed model and its potential for large-scale analysis of fiber-reinforced composites.

1. Introduction

Fiber composites have been widely used in modern industries due to their unique mechanical, thermal and electrical properties. In order to have a deep understanding of the reinforcement mechanism of fibers and to help establish efficient fabrication processes, much effort has been made on the study of the relations between microstructures and overall properties of fiber composites. As a boundary-type numerical method, the boundary element method (BEM) is particularly suitable for the modeling and numerical analysis of fiber composites since only the outer surface of the matrix and the matrix-fiber interfaces need to be meshed. This feature significantly reduces the meshing complexity of the fiber composites compared with other volume-type numerical methods. In addition, detailed distributions of physical variables such as stresses, temperature or electrical potentials along the matrix-fiber interfaces are readily obtained for further investigations on the reinforcement path of fibers. Numerical study on the modeling and analysis of fiber composites by the BEM has been continuously reported [1–9].

Conventionally, the cost of storing the coefficient matrix arising from the BEM is approximately $O(N^2)$, where N is the number of unknowns. This requirement makes the solution of the corresponding linear equation system by standard direct of iterative algorithms significantly time-consuming. In order to improve efficiency of the BEM, several fast algorithms have been extended in this field to achieve fast and large-scale BEM solutions. Of particular interest is the fast multipole method (FMM) [10–14]. This algorithm was initially proposed for the fast solution of potential problems with $O(N)$ scales by novel tree-structure-based operations. Extensive researches on the applications of the fast multipole BEM (FMBEM) in the fields of electro-magnetics, acoustics and elasticity have been reported during the last twenty years. A comprehensive review of the FMBEM can be found in the literature [15].

For the fast BEM modeling of fiber composites or similar micro-heterogeneous materials, the FMBEM is the most reported fast algorithm for the two- and three-dimensional elastic/thermal studies [16–26]. Basically, standard multi-domain BEM formulations are available to treat such inclusion problems. Significant meshing simplification can also be achieved when inclusions or holes have very simple shapes such as circles [27]. On the other hand, fibers have special structural or physical properties that may be used to simplify the associated BEM formulation. Commonly, fibers can be treated as rigid inclusions due to their much higher values of stiffness or thermal conductivity than those of the matrix. This rigid inclusion model makes the FMBEM very efficient when treating carbon nanotube composites [16,18,21,23]. If the variations of fiber properties are of interest, a repeated similar

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0955-7997/$ - see front matter
http://dx.doi.org/10.1016/j.enganabound.2012.11.007

sub-domain model can be used instead of the rigid inclusion model to simplify the mathematical treatment of fibers with the same shape [17,20]. However, three-dimensional fibers in the above-mentioned literatures are meshed with surface elements. This meshing requirement significantly increases the number of unknowns for a fiber when it has a high value of aspect ratio. Therefore, modeling sufficient number of long fibers becomes difficult due to a huge number of unknowns generated compared with those of short fibers. In addition, fibers with very large aspect ratios result in an ill-conditioned coefficient matrix due to very closed boundary points located on the opposite sides of a fiber. An efficient way to overcome this problem is to make the fiber radius become zero and to apply the dual BEM formulation to avoid the ill-conditioning [28–31]. One can notice that a long fiber has its physical variables (stresses or temperature) remaining approximately constant over the cross-sectional area compared with their distributions along the fiber axial direction. This feature may be used for further simplification of the long fiber meshing and allow for significantly increased number of fibers to be modeled with common computer resources.

In this paper, a line inclusion model is introduced as a representative of the fiber for the thermal analysis of fiber composites by the BEM. This model treats fibers as lines with assigned circular cross-sections as the property. This assumption is valid when a fiber has a large aspect ratio. Therefore, fiber temperature varies only along the length direction and fibers can be meshed with line elements. Compared with conventional surface meshing, this simplified approach can increase the number of fibers to be analyzed at least one order of magnitude higher. For each line element, the boundary integral happens on the associated cylindrical surface in cylindrical coordinates. In order to effectively treat fibers with arbitrary lengths, an interface integral BEM initially proposed by Gao [32] is extended herein to govern this line-inclusion problem with a single-domain boundary integral equation. The FMM is adopted as the fast BEM solver for the large-scale fiber composite problems. In the numerical examples, validity and large-scale efficiency of the proposed method for the thermal analysis of fiber composites are demonstrated.

2. Interface integral BEM for line-inclusion problems

The boundary integral equation (BIE) governing the steady-state heat conduction in a three-dimensional homogenous domain is expressed as,

$$c(x)T(x)+\int_S \frac{\partial G^*(x,y)}{\partial n}T(y)dS(y)=\int_S G^*(x,y)\left[-\frac{1}{k}q(y)\right]dS(y) \tag{1}$$

where x and y denote the source and field points at the boundary S, respectively; T and q are the boundary temperature and heat flux, respectively; n is the outward normal to the boundary S; k is the heat conductivity; $c(x)$ is 0.5 for smooth boundaries; $G^*(x,y)$ is the kernel function for the three-dimensional heat conduction problem defined as,

$$G^*(x,y)=\frac{1}{4\pi r} \tag{2}$$

with r denoting the distance between x and y. For the heat conduction in multi-domains, Gao proposed an interface integral BEM for governing this problem with a single-domain BIE formulation given by [32],

$$\hat{k}(x)T(x)+\sum_i \int_{S_i^0} k_i \frac{\partial G^*(x,y)}{\partial n}T(y)dS(y)+\sum_{i,j}^{i\neq j}\int_{S_{ij}^I}\Delta k\frac{\partial G^*(x,y)}{\partial n'}T(y)dS(y)$$

$$=-\sum_i \int_{S_i^0} G^*(x,y)q(y)dS(y) \tag{3}$$

where S_i^0 is the outer boundary of the i-th domain denoted by V_i, $S_{i,j}^I$ the interface of the i-th and j-th domains, k_i the heat conductivity of the i-th domain, n' the normal direction to the interface $S_{i,j}^I$ pointing from the i-th to the j-th domain. $\hat{k}(x)$ and Δk are given by [32],

$$\hat{k}(x)=\begin{cases}\frac{k_i}{2}, & x\in S_i^0\\ \frac{k_i+k_j}{2}, & x\in S_{i,j}^I\\ k_i, x\in V_i\end{cases} \tag{4}$$

$$\Delta k = k_i - k_j \tag{5}$$

provided x is on the smooth boundaries. The advantage of Eq. (3) is that integrals on both the outer boundaries and interfaces are covered by a single equation formulation with the interfacial continuity conditions automatically satisfied. As fiber composites are commonly treated as two-phase structures, thermal analysis of such special multi-domain problems can be achieved with a simplified format of Eq. (3). Fig. 1 shows a model as a representative element of the fiber composites. Let V^0 and V^I denote the matrix and fiber domains, respectively, S^0 and S^I the outer boundaries of the matrix and the fiber-matrix interfaces, respectively. S^I is assumed to be perfectly bonded. By defining new variables $\hat{T}$ and $\hat{q}$ as,

$$\hat{T}\left(x\right)=\begin{cases}k_0T(x), & x\in S^0\\ (k_I-k_0)T\left(x\right), & x\in V^I\cup S^I\end{cases}$$

$$\hat{q}(x)=\begin{cases}-q(x), & x\in S^0\\ 0, & x\in S^I\end{cases} \tag{6}$$

Eq. (3) is rewritten for governing fiber composite problems as,

$$\hat{c}(x)\hat{T}(x)+\int_{S^0+S^I}\frac{\partial G^*(x,y)}{\partial n}\hat{T}(y)dS(y)=\int_{S^0+S^I}G^*(x,y)\hat{q}(y)dS(y) \tag{7}$$

where k_0 and k_I are heat conductivities of the matrix and fibers, respectively. $\hat{c}(x)$ is defined as,

$$\hat{c}(x)=\begin{cases}\frac{1}{2}, & x\in S^0\\ \frac{1}{2}\frac{k_I+k_0}{k_I-k_0}, & x\in S^I\\ \frac{k_I}{k_I-k_0}, & x\in V^I\end{cases} \tag{8}$$

Eq. (7) has the same formulation as Eq. (1) and can be solved by the standard single-domain BEM. According to Eq. (6), only the derivative of G^* needs to be integrated on S^I. After $\hat{T}$ and $\hat{q}$ are obtained by the BEM, T and q are readily derived with Eq. (6).

In order to discretize Eq. (7) we use collocation method and piece-wise constant boundary element. The triangular element has been reported as a widely used element for the discretization of the matrix-fiber interfaces [18–20,23]. For the long fibers, meshing with this kind of element typically generates a large number of unknowns, thus having a significant limitation on the total fiber count to be analyzed. In fact, temperature over the

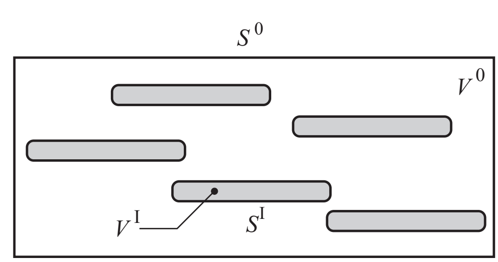

Fig. 1. Representative element of fiber composites.