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Construction on two-level factorials with flexible partially replicated runs via quarter foldover

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ABSTRACT

Fractional factorial designs with partially replicated runs are desirable, since they not only save experimental cost and but also estimate the experimental error variance. In this paper, a simple technique called as quarter foldover, which can construct two-level factorial designs with flexible partially replicated runs, is firstly proposed. For an unreplicated initial design with resolution III (IV) or higher, whether regular or not, the sufficient and necessary conditions for the constructed design to be a resolution III (IV) or higher design with flexible partially replicated runs are investigated by the tool of indicator function. Theoretical results and examples show that the constructed designs are more flexible than most of the existing results in terms of the replicated runs. For initial designs of 12, 16, 20 and 24 runs with s factors, where $3 \leq s \leq 6$, a catalog of optimal plans to construct some designs with highest resolution and flexible partially replicated runs is also tabulated for practice.

1. Introduction

At the early experiment stages, given cost-effectiveness, unreplicated factorial designs are commonly used to identify important or active factorial effects. But such unreplicated experiments are a big challenge to statistical inference, because no pure replicates can not estimate the experimental error variance. A simple approach to obtain pure replicates is the fully replicated design, but the method often leads to the surge of experimental cost. Thus designs with partial replicates are optimal alternatives, because they are not only more powerful than the unreplicated design, but also provide cost savings compared to the fully replicated design. The feasibility and effectiveness of partially replicated designs are verified by Liao and Chai (2009).

There are considerable interests in studying factorial designs with partial replication. Dykstra (1959) proposed some high-resolution designs including repeated runs. Pigeon and McAllister (1989) showed that designs with partial replication have done an unbiased estimate for the error variance without sacrificing the orthogonality of main effects. However, they did not demonstrate how to construct the designs for the general case. Lupinaci and Pigeon (2008) proposed a new class of partially replicated orthogonal main-effect plans based on the Hadamard designs, in which the number of repeated runs is half of the initial designs. Liau (2008) extended one of examples in Pigeon and McAllister (1989) to obtain a general method of constructing two-level designs with partial replication by two techniques. Dasgupta et al. (2010) provided some partially replicated 16-run designs with m factors whose replicated runs are 4, where $4 \leq m \leq 11$. Liao and Chai (2004) proposed a systematic method for constructing two-level factorial designs of user-specified resolution with partial duplication from

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parallel flats designs. Subsequently, Liao and Chai (2009) considered the construction of parallel-flats designs with two identical parallel flats that allow estimation of a set of specified possibly active effects and the pure error variance. However, the number of repeated runs in the designs provided by Liao and Chai (2004, 2009) must be a power of 2. Tsai and Liao (2011) and Tsai et al. (2012) extended the results in Liao and Chai (2004, 2009) to regular and non-regular designs. In view of extended minimum aberration criterion, partially replicated designs constructed through two-level orthogonal arrays were proposed in Tsai and Liao (2014). By using of semifoldover and indicator function, Ou et al. (2013) provided a class partially replicated designs, in which the number of repeated runs was half of the initial designs.

Foldover is a useful technique to de-alias effects, the (full) foldover design is obtained by reversing the signs of one or more factors in the initial design and adding the new runs to the initial design. Taking experimental cost into account, a partial foldover is desirable, and semifoldover is regarded as an effective technique for this need. Semifoldover designs are obtained by reversing signs of one or more factors in the initial design, but only half of the new runs is selected. Thus, semifoldover designs are more valuable than full foldover designs in practice. Readers can refer to Mee and Peralta (2000), Huang et al. (2008), Balakrishnan and Yang (2009, 2011) and Ou et al. (2013).

In this paper, stemming from the semifoldover method in Ou et al. (2013), we extend the results beyond the above limitations of the existing partially replicated designs and obtain a class of partially replicated designs via quarter foldover (defined in Section 2) and indicator function. It is interesting that the resulting designs have high resolution and are more flexible in terms of the replicated runs.

The paper is organized as follows. In Section 2, some concepts related to indicator function of two-level factorials are introduced, and a simple technique for selecting runs called as quarter foldover is defined and described through indicator function. In Section 3, the construction on two-level factorial design with flexible partially replicated runs is provided via quarter foldover, and indicator function of the constructed design is obtained. For an unreplicated initial design with resolution III.a (IV.a), where a is a positive integer, the sufficient and necessary conditions for the constructed design to be a resolution III (IV) or higher design with flexible partially replicated runs are respectively investigated in Sections 4 and 5 by the tool of indicator function. Some illustrative examples are also given in Sections 4 and 5 to lend further support for the theoretical results. In Section 6, some optimal plans to construct the designs with flexible partially replicated runs are presented for practice. Finally, we make some concluding remarks in Section 7.

2. Indicator function and quarter foldover

Suppose $\mathscr{D}$ is a full two-level factorial design with s factors, i.e., $\mathscr{D}$ contains 2^s level combinations (or runs) $x = (x_1, \ldots, x_s)$, where $x_i = -1$ or 1, $i = 1, 2, \ldots, s$. Let $\mathscr{F}$ be a subset of $\mathscr{D}$. The indicator function of $\mathscr{F}$ is a function $f(x)$ defined on $\mathscr{D}$ such that $f(x) = \begin{cases} 1, & \text{if } x \in \mathscr{F}, \\ 0, & \text{if } x \in \mathscr{D} - \mathscr{F}. \end{cases}$ For applications of the indicator function, one can refer to Balakrishnan and Yang (2006a,b, 2009, 2011), Ou and Qin (2010) and Ou et al. (2013).

Denote $L = \{\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_s) | \alpha_i = 1 \text{ or } 0 \text{ , for } i = 1, 2, \ldots, s\}$, and $x^\alpha = x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_s^{\alpha_s}$ $(\alpha \neq 0)$ which represents a main effect or an interaction. Following Fontana et al. (2000), the indicator function $f(x)$ of $\mathscr{F}$ can be uniquely represented by the polynomial function defined on $\mathscr{D}$ as

$$f(x) = \sum_{\alpha \in L} b_\alpha x^\alpha. \tag{1}$$

The coefficient b_α in (1) can be determined by the formula $b_\alpha = \frac{1}{2^s} \sum_{x \in \mathscr{F}} x^\alpha$. In particular, $b_0 = N/2^s$, where N is the number of runs of $\mathscr{F}$, i.e., b_0 is equal to the ratio between the number of runs of $\mathscr{F}$ and the number of runs of $\mathscr{D}$. Usually, for an unreplicated fractional factorial design, $|b_\alpha / b_0| \leq 1$. Fontana et al. (2000) and Ye (2003) indicated that $\mathscr{F}$ is regular if and only if $|b_\alpha / b_0| = 1$ for any $b_\alpha \neq 0$.

Note that x^α in each term with non-zero coefficient (except the constant) in the indicator function $f(x)$ is called as a word of $\mathscr{F}$, and the length of a word x^α is computed by the formula $\|x^\alpha\| = \|\alpha\| + (1 - |b_\alpha / b_0|)$, where $\|\alpha\| = \sum_{i=1}^s \alpha_i$ represents the number of factors in the word x^α (Li et al., 2003). The shortest wordlength of $f(x)$ is called as the generalized resolution of $\mathscr{F}$ and denoted by $R(\mathscr{F})$.

Let $\mathscr{F}(r)$ be the foldover design of $\mathscr{F}$, which is obtained by reversing the signs of r factors $x_{i_1}, x_{i_2}, \ldots, x_{i_r}$ in the initial design $\mathscr{F}$, $1 \leq r \leq s$. Denote by $\mathscr{W}_0^r$ the set of all x^α in (1) contains 0 or even number of the r foldover factors, and $\mathscr{W}_1^r$ the set of all x^α in (1) contains odd number of the r foldover factors. Then (1) can be rewritten as

$$f(x) = \sum_{x^\alpha \in \mathscr{W}_0^r} b_\alpha x^\alpha + \sum_{x^\alpha \in \mathscr{W}_1^r} b_\alpha x^\alpha. \tag{2}$$

Following Li et al. (2003), the indicator function of $\mathscr{F}(r)$, denoted by $f_1(x)$, can be written as

$$f_1(x) = \sum_{x^\alpha \in \mathscr{W}_0^r} b_\alpha x^\alpha - \sum_{x^\alpha \in \mathscr{W}_1^r} b_\alpha x^\alpha. \tag{3}$$

Let

$$\mathscr{F}_{z_1=e_1, z_2=e_2} = \{x \in \mathscr{F} | z_1 = e_1, z_2 = e_2\},$$