



Remarks on limit theorems for reversible Markov processes and their applications

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ABSTRACT

We propose some backward–forward martingale decompositions for functions of reversible Markov chains. These decompositions are used to prove the functional Central limit theorem for reversible Markov chains with asymptotically linear variance of partial sums. We also provide a proof of the equivalence between asymptotic linearity of the variance and convergence of the integral of $1/(1-t)$ with respect to the associated spectral measure ρ . We show a result on uniform integrability of the supremum of the average sum of squares of martingale differences. We also study the asymptotic behavior of linear processes having as innovations mean zero square integrable functions of stationary reversible Markov chains. We include in our study the long-range dependence case. We apply this study to several cases of reversible stationary Markov chains that arise in regression estimation.

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1. Introduction

An important theoretical question with numerous practical implications is to prove stability of the central limit theorem under formation of linear sums. By this we understand that if $S_n(\xi)/\sqrt{n} = \sum_{i=1}^n \xi_i/\sqrt{n}$ converges in distribution to a normal variable, then the same result holds for $S_n(X)$ properly normalized, where $(X_i, 1 \leq i \leq n)$ are linear functions of $(\xi_i, -\infty < i < \infty)$. This problem was first studied in the literature by Ibragimov (1962), who proved that if $(\xi_i, i \in \mathbb{Z})$ are centered i.i.d. with finite second moments, $S_n(X)/b_n$ satisfies the central limit theorem (CLT). The extra condition of finite second moment was removed by Peligrad and Sang (2013). The central limit theorem for $S_n(X)/b_n$ for the case when the innovations are square integrable martingale differences was proved by Peligrad and Utev (1997, 2006), where an extension to generalized martingales was also given.

On the other hand, motivated by applications to unit root testing and to isotonic regression, a related question is to study the limiting behavior of $S_{[nt]}(X)/b_n$ (here and throughout the paper $[x]$ denotes the integer part of x). The first results on this topic are due to Davydov (1970), who established convergence to fractional Brownian motion for the case of i.i.d. innovations $(\xi_i, 1 \leq i \leq n)$. Extensions to dependent settings under certain projection criteria can be found for instance in Wu and Min (2005) and Dedecker et al. (2011), among others.

Kipnis and Varadhan (1986) considered partial sums $S_n(X)$ of an additive mean zero functional of a stationary reversible ergodic Markov chain and showed that the convergence of $\text{var}(S_n)/n(X)$ implies convergence of $\{S_{[nt]}(X)/\sqrt{n}, 0 \leq t \leq 1\}$ to

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the Brownian motion. There is a considerable number of papers that further extend and apply this result to infinite particle systems, random walks, processes in random media, Metropolis–Hastings algorithms. Among others, [Kipnis and Landim \(1999\)](#) considered interacting particle systems, [Tierney \(1994\)](#) discussed the applications to Markov Chain Monte Carlo and [Wu \(1999\)](#) studied the law of the iterated logarithm. Here, we will consider other cases of linear processes such as the causal model, applications to kernel estimation and linear regression.

We review the central limit theorem for stationary Markov chains with self-adjoint operator and general state space. We investigate the case when the variance of the partial sum is asymptotically linear in n , and propose a new proof of the functional CLT for ergodic reversible Markov chains in Corollary 1.5 of [Kipnis and Varadhan \(1986\)](#). We prove the equivalence of $\lim_{n \rightarrow \infty} \text{var}(S_n(\xi))/n < \infty$ and convergence of $\int_{-1}^1 \frac{\rho(dt)}{1-t}$ for a mean zero function f of a stationary reversible Markov chain. Here, ρ is the spectral measure corresponding to f . This equivalence is used to provide a new forward–backward martingale decomposition for the given class of processes. Among new results of this paper, is a forward–backward martingale decomposition for stationary reversible Markov chains. In [Proposition 2](#), we state a convergence theorem that helps establish a martingale convergence theorem in [Lemma 2](#). A new proof of the central limit theorem based on [Heyde \(1974\)](#) is provided. Throughout this paper we use the spectral theory of bounded self-adjoint operators. In [Section 1](#) we have the introduction, [Section 2](#) is about the forward–backward martingale decomposition and [Section 3](#) tackles the new proof of the functional central limit theorem for ergodic reversible Markov chains and [Section 4](#) provides applications to various statistical models.

1.1. Definitions and notations

We assume that $(\gamma_n)_{n \in \mathbb{Z}}$ is a stationary reversible Markov chain defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with values in a general state space $(S, \mathcal{A})$. The marginal distribution is denoted by $\pi(A) = \mathbb{P}(\gamma_0 \in A)$. Assume that there is a regular conditional distribution for γ_1 given γ_0 denoted by

$$Q(x, A) = \mathbb{P}(\gamma_1 \in A | \gamma_0 = x).$$

Let Q also denote the Markov operator acting via

$$(Qg)(x) = \int_S g(s)Q(x, ds).$$

Next, let $\mathbb{L}_0^2(\pi)$ be the set of measurable functions on S such that $\int g^2 d\pi < \infty$ and $\int g d\pi = 0$. If $g, h \in \mathbb{L}_0^2(\pi)$, the integral $\int_S g(s)h(s)d\pi$ will sometimes be denoted by $\langle g, h \rangle$.

For some function $g \in \mathbb{L}_0^2(\pi)$, let

$$\xi_i = g(\gamma_i), \quad S_n(\xi) = \sum_{i=1}^n \xi_i, \quad \sigma_n(g) = (\mathbb{E}S_n^2(\xi))^{1/2}. \quad (1)$$

Denote by $\mathcal{F}_k$ the σ -field generated by γ_i with $i \leq k$ and by $\mathcal{I}$ the invariant σ -field.

For any integrable random variable X we denote $\mathbb{E}_k X = \mathbb{E}(X | \mathcal{F}_k)$. With this notation, $\mathbb{E}_0 \xi_1 = Qg(\gamma_0) = \mathbb{E}(\xi_1 | \gamma_0)$. We denote by $\|X\|_p$ the norm in $\mathbb{L}^p(\Omega, \mathcal{F}, \mathbb{P})$.

The Markov chain is called reversible if $Q = Q^*$, where Q^* is the adjoint operator of Q . In this setting, the condition of reversibility is equivalent to requiring that (γ_0, γ_1) and (γ_1, γ_0) have the same distribution. Equivalently

$$\int_A Q(\omega, B)\pi(d\omega) = \int_B Q(\omega, A)\pi(d\omega)$$

for all Borel sets $A, B \in \mathcal{A}$. The spectral measure of Q with respect to g is concentrated on $[-1, 1]$ and will be denoted by ρ_g . Then

$$\mathbb{E}(Q^m g(\gamma_0) Q^n g(\gamma_0)) = \langle Q^m g, Q^n g \rangle = \int_{-1}^1 t^{n+m} \rho_g(dt).$$

We denote by $W(t)$ the standard Brownian motion. All throughout the paper $\implies$ denotes convergence in distribution, $\xrightarrow{P}$ denotes convergence in probability and $[x]$ is the integer part of x .

We also need to introduce here some very useful notions from the spectral theory.

1.2. Spectral Theory of self-adjoint operators

Self-adjoint operators have spectral families with certain regularity properties, beyond the properties shared by all spectral families, which are very important in the proof of the theorems in this paper. Recall that a linear vector space $\mathbb{H}$ is a Hilbert space, if it is endowed with an inner product $\langle \cdot, \cdot \rangle$, associated with a norm $\|\cdot\|$ and metric $d(\cdot, \cdot)$, such that every Cauchy sequence has a limit in $\mathbb{H}$. Elements x, y of a Hilbert space are said to be orthogonal if $\langle x, y \rangle = 0$. Suppose there is a non-decreasing family $(M(\lambda), \lambda \in \mathbb{R})$ of closed subspaces of $\mathbb{H}$ depending on a real parameter λ , such that the intersection

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