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Multivariate nonparametric estimation of the Pickands dependence function using Bernstein polynomials

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ABSTRACT

Many applications in risk analysis require the estimation of the dependence among multivariate maxima, especially in environmental sciences. Such dependence can be described by the Pickands dependence function of the underlying extreme-value copula. Here, a nonparametric estimator is constructed as the sample equivalent of a multivariate extension of the madogram. Shape constraints on the family of Pickands dependence functions are taken into account by means of a representation in terms of Bernstein polynomials. The large-sample theory of the estimator is developed and its finite-sample performance is evaluated with a simulation study. The approach is illustrated with a dataset of weekly maxima of hourly rainfall in France recorded from 1993 to 2011 at various weather stations all over the country. The stations are grouped into clusters of seven stations, where our interest is in the extremal dependence within each cluster.

1. Introduction and background

In recent years, inference methods for assessing the extremal dependence have been increasingly in demand. This is particularly due to growing requests for the analysis of multivariate extreme values in the fields of environmental and economic sciences, where the dimension of the random vector under study is often greater than 2. For example, Fig. 1 displays a map of clusters containing seven weather stations in France each; see Bernard et al. (2013) for details on the construction of the clusters. The data consist of weekly maxima of hourly rainfall recorded at each station.[1] It would be of interest to hydrologists to infer the dependence within each of the seven-dimensional vectors of component-wise maxima and to compare the dependence structures among clusters. Such an endeavor represents the main motivation of this work.

Let $\boldsymbol{X} = (X_1, \ldots, X_d)$ be a d-dimensional ($d \geq 2$) random vector of maxima that follows a multivariate max-stable distribution G; for more background on univariate and multivariate extreme-value theory, see for instance Beirlant et al. (2004), de Haan and Ferreira (2006), or Falk et al. (2010). The margins of G, denoted by $F_i(x) = \mathbb{P}\{X_i \leq x\}$, for all $x \in \mathbb{R}$ and

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1 Data provided by Météo-France and published within the R package ClusterMax, freely available from the homepage of Philippe Naveau, http://www.lsce.ipsl.fr/Pisp/philippe.naveau/.

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0378-3758/

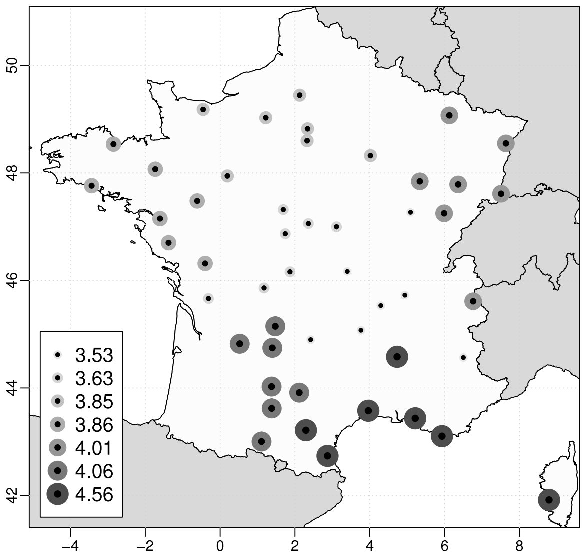

Fig. 1. Analysis of French weekly precipitation maxima in the period 1993–2011. Clusters of 49 weather stations and their estimated extremal coefficients in dimension $d = 7$ obtained with the projected version of the madogram estimator, see Section 5 for details.

$i = 1, \ldots, d$, are univariate max-stable distributions. The joint distribution takes the form

$$G(\boldsymbol{x}) = C\big(F_1(x_1), \ldots, F_d(x_d)\big), \quad \boldsymbol{x} \in \mathbb{R}^d, \tag{1.1}$$

where C is an extreme-value copula:

$$C(u_1, \ldots, u_d) = \exp\big(-\ell(-\log u_1, \ldots, -\log u_d)\big), \quad \boldsymbol{u} \in (0, 1]^d, \tag{1.2}$$

with $\ell : [0, \infty)^d \to [0, \infty)$ the so-called stable tail dependence function. The latter function is homogeneous of order one and is therefore determined by its restriction on the unit simplex, the restriction itself being called the Pickands dependence function, denoted here by A. Formally, we have

$$\ell(\boldsymbol{z}) = (z_1 + \cdots + z_d)\, A(\boldsymbol{w}), \quad \boldsymbol{z} \in [0, \infty)^d, \tag{1.3}$$

where $\boldsymbol{w} = (w_1, \ldots, w_d)$, $w_i = z_i/(z_1 + \cdots + z_d)$ for $i = 1, \ldots, d-1$ and $w_d = 1 - w_1 - \cdots - w_{d-1}$. We view A as a function defined on the $(d-1)$-dimensional unit simplex

$$\mathcal{S}_{d-1} := \left\{ (w_1, \ldots, w_d) \in [0, 1]^d : \sum_{i=1}^{d} w_i = 1 \right\}. \tag{1.4}$$

Let $\mathcal{A}$ be the family of functions $A : \mathcal{S}_{d-1} \to [1/d, 1]$ that satisfy the following conditions:

(C1) $A(\boldsymbol{w})$ is convex, i.e., $A(a\boldsymbol{w}_1 + (1-a)\boldsymbol{w}_2) \le aA(\boldsymbol{w}_1) + (1-a)A(\boldsymbol{w}_2)$, for $a \in [0, 1]$ and $\boldsymbol{w}_1, \boldsymbol{w}_2 \in \mathcal{S}_{d-1}$;
(C2) $A(\boldsymbol{w})$ has lower and upper bounds

$$1/d \le \max(w_1, \ldots, w_{d-1}, w_d) \le A(\boldsymbol{w}) \le 1,$$

for any $\boldsymbol{w} \in \mathcal{S}_{d-1}$.

Any Pickands dependence function belongs to the class $\mathcal{A}$ (Falk et al., 2010, Ch. 4). The converse is not true, however; see Beirlant et al. (2004, p. 257) for a counterexample. A characterization of the class of stable tail dependence functions has been given in Ressel (2013). In condition (C2), the lower and upper bounds represent the cases of complete dependence and independence, respectively.

Many parametric models have been introduced for modeling the extremal dependence for a variety of applications, with summaries to be found in Kotz and Nadarajah (2000) and Padoan (2013). However, such finite-dimensional parametric models can never cover the full class of Pickands dependence functions. For this reason, several nonparametric estimators of the Pickands dependence function have been proposed: see for instance Pickands (1981), Capéraà et al. (1997), Hall and Tajvidi (2000), Zhang et al. (2008), Genest and Segers (2009), Bücher et al. (2011), Gudendorf and Segers (2011, 2012), and Berghaus et al. (2013). All of these estimators require further adjustments to ensure they are genuine Pickands dependence functions.

Given an independent random sample from a multivariate distribution with continuous margins and whose copula is an extreme-value copula, we propose a nonparametric estimator for its Pickands dependence function. In the bivariate case,