Statistics and Probability Letters 129 (2017) 34–41

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

Hölder continuity for stochastic fractional heat equation with colored noise

Kexue Li

School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an 710049, China

ARTICLE INFO

Article history:
Received 15 November 2016
Received in revised form 21 April 2017
Accepted 23 April 2017
Available online 28 April 2017

MSC:
35K55
35K08

Keywords:
Stochastic fractional heat equation
Fractional heat kernel
Colored noise
Hölder continuity

ABSTRACT

In this paper, we consider semilinear stochastic fractional heat equation $\frac{\partial u_t}{\partial t} = -(-\triangle)^{\beta/2}u_t + \sigma(u_t)\dot{\eta}$. The Gaussian noise $\dot{\eta}$ is assumed to be colored in space with covariance of the form $E(\dot{\eta}(t,x)\dot{\eta}(s,y)) = \delta_0(t-s)f_\alpha(x-y)$, where f_α is the Riesz kernel $f_\alpha(x) \propto |x|^{-\alpha}$. We obtain the spatial and temporal Hölder continuity of the mild solution.
© 2017 Elsevier B.V. All rights reserved.

1. Introduction

In this paper, we consider the following stochastic fractional heat equation

$$\begin{cases} \dfrac{\partial u_t}{\partial t} = -(-\Delta)^{\beta/2}u_t + \sigma(u_t)\dot{\eta}, \quad t \geq 0,\ x \in \mathbb{R}^d, \\ u_0(x) = \phi(x), \end{cases} \tag{1.1}$$

where $0 < \beta \leq 2$, $(-\Delta)^{\beta/2}$ denotes the fractional Laplacian defined by the Fourier transform

$$(\mathcal{F}(-\triangle)^{\beta/2}\varphi)(\xi) = (2\pi|\xi|)^\beta \mathcal{F}(\varphi)(\xi), \quad \varphi \in C_0^\infty(\mathbb{R}^d),$$

here $\mathcal{F}$ denotes the Fourier transform,

$$(\mathcal{F}\varphi)(\xi) = \int_{\mathbb{R}} e^{-2i\pi\xi\cdot x}\varphi(x)dx. \tag{1.2}$$

$\dot{\eta}$ is the Gaussian space time colored noise with covariance of the form

$$E[\dot{\eta}(t,x)\dot{\eta}(s,y)] = \delta_0(t-s)f_\alpha(x-y), \tag{1.3}$$

E-mail address: kexueli@gmail.com.

http://dx.doi.org/10.1016/j.spl.2017.04.020
0167-7152/© 2017 Elsevier B.V. All rights reserved.

where $\delta_0(\cdot)$ denotes the Dirac delta function and (Dalang, 1999, Ex.1)

$$f_\alpha(x) = c_{\alpha,d} g_\alpha(x) = (\mathcal{F} g_{d-\alpha})(x), \; g_\alpha(x) = \frac{1}{|x|^\alpha}, \quad \alpha \in (0, d), \tag{1.4}$$

where $c_{\alpha,d} > 0$ is a constant depending only on α and d (see Stein, 1970, Chap.V, Section 1, Lemma 2(a)).

We assume that the following conditions hold:

(A1) ϕ is bounded and ρ-Hölder continuous.

(A2) σ is Lipschitz continuous and there exists a constant K such that $|\sigma(x) - \sigma(y)| \le K|x - y|$ and $|\sigma(x)| \le K(1 + |x|)$.

The mild solutions of (1.1) are the solutions of the integral equation

$$u_t(y) = (u_0 * p_t)(y) + \int_0^t \int_{\mathbb{R}^d} p_{t-s}(x - y)\sigma(u_s(x))\eta(ds, dx) \tag{1.5}$$

where the fractional heat kernel $p_t(x)$ is the fundamental solution of

$$\frac{\partial v}{\partial t} = -(-\Delta)^{\beta/2} v, \tag{1.6}$$

and $*$ denotes the usual convolution operator, $(f * g)(x) = \int_{\mathbb{R}^d} f(x - y)g(y)dy$. We impose $\alpha < d \wedge \beta$ to guarantee the existence and uniqueness of the mild solution of (1.1) (see, e.g., Foondun and Khoshnevisan, 2013; Ferrante and Sanz-Solé, 2006; Liu et al., 2017). It is known that $p_t(x)$ satisfies the following inequality (Jakubowski and Serafin, 2016; Chen et al., 2012; Bogdan and Jakubowski, 2007)

$$\frac{c_1 t}{(t^{1/\beta} + |x|)^{d+\beta}} \le p_t(x) \le \frac{c_2 t}{(t^{1/\beta} + |x|)^{d+\beta}}, \tag{1.7}$$

where $t > 0$, $x \in \mathbb{R}^d$, c_1 and c_2 are positive constants depending on β and d.

Stochastic PDEs with colored noise have been studied in many papers (see, e.g., Balan and Conus, 2016; Balan and Tudor, 2010; Conus et al., 2013; Dalang, 1999). Recent years, Hölder continuity for stochastic parabolic equations has attracted much attention, see Hu et al. (2013) and Cui et al. (2016), for example. Sanz-Solé and Sarrà (1999) studied the Hölder continuity in time and space of the solution of the stochastic heat equation on $\mathbb{R}_+ \times \mathbb{R}^d$:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u + \sigma(u)\dot{W}, \quad t \ge 0, \; x \in \mathbb{R}^d, \\ u(0, x) = \phi(x), \end{cases} \tag{1.8}$$

where the initial condition ϕ is a bounded ρ-Hölder continuous function for some $\rho \in (0, 1)$, the Gaussian noise $\dot{W}$ with covariance given by

$$E[\dot{W}(t, x)\dot{W}(s, y)] = \delta_0(t - s)f(x - y),$$

when f is the Fourier transform of a tempered measure μ on $\mathbb{R}^d$. Theorem 2.1 of Sanz-Solé and Sarrà (1999) shows that, if the measure μ satisfies the condition

$$\int_{\mathbb{R}^d} \left(\frac{1}{1 + |\xi|^2} \right)^\theta \mu(d\xi) < \infty \quad \text{for some } \theta \in (0, 1),$$

then the solution u of Eq. (1.8) has a Hölder-continuous modification of order smaller than $\frac{1}{2}(\rho \wedge (1-\theta))$ in time and smaller than $\rho \wedge (1 - \theta)$ in space. When $f(x) = c_{\alpha,d}|x|^{-\alpha}$ is the Riesz kernel of index $\alpha \in (0, d)$, the solution u of Eq. (1.8) has a Hölder-continuous modification of order smaller than $\frac{1}{2}(\rho \wedge (1 - \alpha/2))$ in time and smaller than $\rho \wedge (1 - \alpha/2)$ in space.

Chen and Dalang (2014) studied space–time regularity of the solution of the nonlinear stochastic heat equation in one spatial dimension driven by space–time white noise with the initial condition being a locally finite measure:

$$\begin{cases} \left(\dfrac{\partial u}{\partial t} - \dfrac{\nu}{2} \dfrac{\partial^2}{\partial x^2} \right) u = \rho(u)\dot{W}, \quad t \in (0, \infty), \; x \in \mathbb{R}, \\ u(0, \cdot) = \mu(\cdot). \end{cases} \tag{1.9}$$

They get the results that on compact sets in which $t > 0$, the classical Hölder-continuity exponents $\frac{1}{4}-$ in time and $\frac{1}{2}-$ in space remain valid. However, on compact sets that include $t = 0$, the Hölder-continuity exponents are $(\frac{\delta}{2} \wedge \frac{1}{4})-$ in time and $(\delta \wedge \frac{1}{2})-$ in space, provided μ is absolutely continuous with an $\delta-$ Hölder continuous density.

Hu et al. (2013) considered the nonlinear stochastic heat equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{1}{2}\Delta u + b(u) + \sigma(u)\dot{W}(t, x), \quad t \ge 0, \; x \in \mathbb{R}^d, \\ u(0, x) = u_0(x), \end{cases} \tag{1.10}$$

Download English Version:

https://daneshyari.com/en/article/5129707

Download Persian Version:

https://daneshyari.com/article/5129707

Daneshyari.com