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Preservation of weak stochastic arrangement increasing under fixed time left-censoring

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ABSTRACT

Both WSAI and CLOAI properties of a random vector are proved to be preserved under left-censoring at fixed times. Applications in threshold default model of financial portfolio selection and system warranty cost allocation are presented as well.

1. Introduction

The monotonicity of random variables based on various stochastic orders is very useful in reliability, financial and actuarial risk as well as other areas related to probability and statistics, and it is well-known that the distribution-free theory of stochastic orders plays an important part in reliability, risk management and utility framework. For a comprehensive dealing with stochastic orders with applications one can refer to monographs of Müller and Stoyan (2002) and Shaked and Shanthikumar (2007). Due to the great recession the dependence among random risks has been paid much attention because sometimes the ignorance of positive dependence may incur hazardous outcome for financial institutes. On the other hand, traditional reliability theory always assumes independence among component lifetimes due to mathematical intractability. However, this is absolutely impractical in most situations because components have to suffer from some common stresses while operating under the same industrial environment. No doubt the model based on the compulsive assumption of independence may be far away from the reality and hence disastrous in extreme occasions. In the wake of requiring high reliability, system safety and financial security, many researchers devote themselves to the study of monotonicity taking into account of statistical dependence among concerned random variables.

Many authors made their effort in the past decade and are still working with stochastic monotonicity of dependent random variables. One group resorts to directly extend the classical stochastic orders between two random variables to the joint stochastic orders so as to incorporate statistical dependence. See, for example, Shanthikumar and Yao (1991), Righter and Shanthikumar (1992), Aly and Kochar (1993) and Belzunce et al. (2016). Also, Pellerey and Zalzadeh (2015) studied relationships between univariate stochastic orders and the corresponding joint stochastic orders. The other group works with stochastic versions arrangement monotonicity of vectors of multiple random variables. See, for example, Cai and Wei (2014, 2015), You and Li (2015), Li and You (2015) and Li and Li (2016) etc. This paper follows the line of the second

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group. As stochastic versions of arrangement increasing, the left tail weakly stochastic arrangement increasing (LWSAI), the weak stochastic arrangement increasing (WSAI) and the conditionally lower orthant arrangement increasing (CLOAI) are found of interesting applications in reliability, policy deductible and coverage limit allocation and asset allocation problems. According to Lemma 5.9 and Proposition 5.7 of Cai and Wei (2015), the WSAI property is preserved when an increasing function is applied to all coordinates and the LWSAI property is preserved under the formation of fixed time left-censoring whenever the thresholds are arrayed in the descending order. It is of both theoretical and practical interest to pursue some different suitable functions preserving WSAI and CLOAI properties when applied to coordinates, respectively. Since left-censoring is popular in lifetime data analysis, in this study we take a step along this line through considering fixed time left-censoring of random variables.

The remaining sections roll out as follows. Section 2 recalls some concerned notions and a technical lemma for ease of reference. We present in Section 3 preservation property of WSAI and CLOAI under the taking of fixed time left-censoring. In Section 4 we develop two simple applications in financial risk and warranty management. For the sake of smoothness, we defer technical proofs of two main results to Appendix.

Throughout this note, all random variables are implicitly assumed to be nonnegative with finite expectations, and the terms *increasing* and *decreasing* mean *nondecreasing* and *nonincreasing*, respectively.

2. Some preliminaries

Set $\mathbb{R}^n = (-\infty, +\infty)^n$, $\mathbb{R}^n_+ = [0, +\infty)^n$, $\mathcal{I}_n = \{1, \ldots, n\}$ and $\mathcal{D}^+_n = \{(x_1, \ldots, x_n) \in \mathbb{R}^n_+ : x_1 \geq \cdots \geq x_n \geq 0\}$. Denote, for $\boldsymbol{x} = (x_1, \ldots, x_n) \in \mathbb{R}^n$ and $1 \leq i < j \leq n$, $\tau_{i,j}(\boldsymbol{x})$ the permutation exchanging x_i and x_j and $\boldsymbol{x}_{\{i,j\}}$ the sub-vector with x_i and x_j deleted.

A real function g is said to be *arrangement increasing* (AI) if $g(\boldsymbol{x}) \geq g(\tau_{i,j}(\boldsymbol{x}))$ for any $\boldsymbol{x} \in \mathbb{R}^n$ and $1 \leq i < j \leq n$ such that $x_i \leq x_j$. For $1 \leq i < j \leq n$, denote $\Delta_{i,j} g(\boldsymbol{x}) = g(\boldsymbol{x}) - g(\tau_{i,j}(\boldsymbol{x}))$ and set

$$\begin{aligned}
\mathcal{A}^{i,j}_{s}(n) &= \left\{g(\boldsymbol{x}) : \Delta_{i,j} g(\boldsymbol{x}) \geq 0 \ \text{ for any } x_j \geq x_i\right\},\\
\mathcal{A}^{i,j}_{lws}(n) &= \left\{g(\boldsymbol{x}) : \Delta_{i,j} g(\boldsymbol{x}) \ \text{ is decreasing in } x_i \in (-\infty, x_j] \text{ for any } x_j\right\},\\
\mathcal{A}^{i,j}_{rws}(n) &= \left\{g(\boldsymbol{x}) : \Delta_{i,j} g(\boldsymbol{x}) \ \text{ is increasing in } x_j \in [x_i, +\infty) \text{ for any } x_i\right\},\\
\mathcal{A}^{i,j}_{ws}(n) &= \left\{g(\boldsymbol{x}) : \Delta_{i,j} g(\boldsymbol{x}) \ \text{ is increasing in } x_j\right\}.
\end{aligned}$$

Cai and Wei (2014, 2015) and Li and Li (2016) introduced the following stochastic versions of AI.

Definition 2.1. A random vector $\boldsymbol{X} = (X_1, \ldots, X_n)$ is said to be

(i) *stochastic arrangement increasing* (SAI) if $\mathrm{E}[g(\boldsymbol{X})] \geq \mathrm{E}[g(\tau_{i,j}(\boldsymbol{X}))]$ for any $g \in \mathcal{A}^{i,j}_{s}(n)$ and $1 \leq i < j \leq n$;
(ii) *left tail weakly stochastic arrangement increasing* (LWSAI) if $\mathrm{E}[g(\boldsymbol{X})] \geq \mathrm{E}[g(\tau_{i,j}(\boldsymbol{X}))]$ for any $g \in \mathcal{A}^{i,j}_{lws}(n)$ and any $1 \leq i < j \leq n$;
(iii) *right tail weakly stochastic arrangement increasing* (RWSAI) if $\mathrm{E}[g(\boldsymbol{X})] \geq \mathrm{E}[g(\tau_{i,j}(\boldsymbol{X}))]$ for any $g \in \mathcal{A}^{i,j}_{rws}(n)$ and any $1 \leq i < j \leq n$;
(iv) *weakly stochastic arrangement increasing* (WSAI) if $\mathrm{E}[g(\boldsymbol{X})] \geq \mathrm{E}[g(\tau_{i,j}(\boldsymbol{X}))]$ for any $g \in \mathcal{A}^{i,j}_{ws}(n)$ and any $1 \leq i < j \leq n$;
(v) *lower orthant arrangement increasing* (LOAI) if its joint distribution function $F(\boldsymbol{x})$ is AI;
(vi) *conditionally lower orthant arrangement increasing* (CLOAI) if $(X_i, X_j) \mid \boldsymbol{X}_{\{i,j\}} = \boldsymbol{x}_{\{i,j\}}$ is LOAI for any $\boldsymbol{x}_{\{i,j\}}$ in support of $\boldsymbol{X}_{\{i,j\}}$ and any $1 \leq i < j \leq n$.

It is easy to verify the following chain of implications:

$$\begin{array}{ccccc}
 & & \text{SAI} & \Longrightarrow & \text{RWSAI}\\
 & & \Downarrow & & \Downarrow\\
\text{CLOAI} & \Longleftarrow & \text{LWSAI} & \Longrightarrow & \text{WSAI}.
\end{array}$$

A random variable X is said to be smaller than the other one Y in the *usual stochastic order* (denoted as $X \leq_{\mathrm{st}} Y$) if $\mathrm{P}(X > t) \leq \mathrm{P}(Y > t)$ for all t, and X is said to be smaller than Y in the *hazard rate order* (denoted as $X \leq_{\mathrm{hr}} Y$) if $\mathrm{P}(Y > t)/\mathrm{P}(X > t)$ increases in t. For more on stochastic orders one may refer to Müller and Stoyan (2002), Shaked and Shanthikumar (2007) and Li and Li (2013).

For random vectors with probability density, LWSAI/RWSAI reduces to the notion of LTPD/UTPD (lower/upper tail permutation decreasing) due to Li and You (2015). Also Cai and Wei (2014, Proposition 5.5) remark that in the context of comonotonicity $X_1 \leq_{\mathrm{st}} \cdots \leq_{\mathrm{st}} X_n$ if and only if $(X_1, \ldots, X_n)$ is SAI. For more on stochastic versions of AI and their applications one may refer to Cai and Wei (2014, 2015), Li and You (2015), You and Li (2015), You et al. (2016), Li et al. (2016) and Li and Li (2016).

As is seen in the following example, there is no necessary implication between CLOAI and WSAI.