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# Designs containing partially clear main effects[☆]

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ABSTRACT

If certain knowledge is available, then additional factors can be studied through designs containing partially clear main effects. Three types are found less useful, and only the fourth type can overcome the drawback. The existence and constructions are studied.



## 1. Introduction

Clear effects criterion is an important rule for selecting designs proposed by Wu and Chen (1992). A main effect or two-factor interaction is said to be clear if it is not aliased with any other main effects or two-factor interactions. Thus clear effects are estimable under the assumption that three-factor and higher-order interactions are negligible, which is assumed throughout this paper. Symmetrical or asymmetrical factorial designs with resolutions III and IV containing clear effects have been studied by Chen and Hedayat (1998), Wu and Wu (2002), Tang et al. (2002), Tang (2006), Ke et al. (2005), Ai and Zhang (2004), Chen et al. (2006), Yang et al. (2006), Yang and Butler (2008), and many others. For a comprehensive review, see Mukerjee and Wu (2006).

If some prior knowledge is available, Lekivetz and Tang (2011) considered the situation in which certain two-factor interactions are assumed to be negligible while other two-factor interactions are not, and constructed the associated two-level robust designs with partially clear two-factor interactions of strength larger than two. In Lekivetz and Tang (2011), a two-factor interaction is said to be partially clear if it is orthogonal to all nonnegligible interactions, but can be aliased with the negligible interactions. Three types of designs with partially clear two-factor interactions are investigated in Lekivetz and Tang. Since all main effects and certain two-factor interactions need to be estimated, designs in Lekivetz and Tang are at least of strength 3. If less runs are desired as cost is limited, then designs of strength 2 with partially clear main effects can be used in this situation. Similarly, a main effect is said to be partially clear if it is orthogonal to nonnegligible two-factor interactions but can be aliased with the negligible two-factor interactions.

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In this paper, orthogonal arrays with partially clear main effects are investigated for two-level regular fractional factorial designs. Hence the run size is restricted to $n = 2^k$. Section 2 introduces the concept of partially clear main effects and four types of them are considered. Three of them are found to be less powerful than orthogonal arrays of strength 3. In Section 3, type 4 designs are found to overcome this drawback and can have more columns than the corresponding strength 3 orthogonal arrays. The existence condition and construction methods of such designs are also obtained. Conclusion is given in Section 4.

## 2. Partially clear main effects

### 2.1. Four types of designs with partially clear main effects

A two-level factorial design $D$ of $n$ runs for $m$ factors is represented by an $n \times m$ matrix of elements $d_{ij} = \pm 1$. Design $D$ becomes a two-level orthogonal array $OA(n, m, 2, t)$ of strength $t$ if, in every $n \times t$ submatrix of $D$, each of $2^t$ level combinations appears with the same frequency. More details can be found in Hedayat et al. (1999). Following Lekivetz and Tang (2011) and Chen and Lin (2016), the factors to be considered in design $D$ are divided into two groups, $G_1 = \{F_1, \ldots, F_{m_1}\}$, containing $m_1$ factors and $G_2 = \{F_{m_1+1}, \ldots, F_{m_1+m_2}\}$, containing $m_2$ factors. Let $G_1 \times G_1$ denote two-factor interactions within the factors in $G_1$, $G_2 \times G_2$ and $G_1 \times G_2$ be defined similarly. Then all two-factor interactions can also be classified into three sets: (1) $G_1 \times G_1$; (2) $G_2 \times G_2$; (3) $G_1 \times G_2$.

Since only main effects need to be estimated in this article, all two-factor interactions are divided into two mutually exclusive and exhaustive parts, $S_1$ and $S_2$, where $S_1$ denotes the nonnegligible two-factor interactions and $S_2$ negligible two-factor interactions. We note that all two-factor interactions have been divided into three mutually exclusive and exhaustive sets in Lekivetz and Tang (2011) as they considered one more set, which means two-factor interactions to be estimated.

Four cases of partially clear main effects plans can be considered in this line:

1. $S_1 = \{G_1 \times G_1, G_2 \times G_2\}, S_2 = \{G_1 \times G_2\}$;
2. $S_1 = \{G_1 \times G_1, G_1 \times G_2\}, S_2 = \{G_2 \times G_2\}$;
3. $S_1 = \{G_1 \times G_2\}, S_2 = \{G_1 \times G_1, G_2 \times G_2\}$;
4. $S_1 = \{G_1 \times G_1\}, S_2 = \{G_1 \times G_2, G_2 \times G_2\}$.

**Remark 1.** Two more cases could be considered: (i) $S_1 = \{G_2 \times G_2\}, S_2 = \{G_1 \times G_1, G_1 \times G_2\}$; (ii) $S_1 = \{G_1 \times G_2, G_2 \times G_2\}, S_1 = \{G_1 \times G_1\}$. However, since some of them are equivalent, we only need to consider the four types above. Let $G_1' = G_2, G_2' = G_1$, then case (i) is equivalent to the above case 4, and case (ii) is equivalent to the above case 2.

From now on, only designs with main effects orthogonal to two-factor interactions in set $S_1$ are considered. For convenience, designs for the four cases are referred to as designs of types 1, 2, 3 and 4, respectively.

We now give a brief introduction of universal optimality. More details can be found in Kiefer (1975), Dey and Suen (2002) and Dey et al. (2005). Let $\mathcal{H}$ be the class of all $N$-run fractional factorial designs for an arbitrary experiment involving $m$ factors, $F_1, \ldots, F_{m_1}, F_{m_1+1}, \ldots, F_{m_1+m_2}$, $m_1 + m_2 = m$. A plan $D \in \mathcal{H}$ is universally optimal if it minimized $\phi(\mathcal{I}_d)$ over $d \in \mathcal{H}$ for each real-valued function $\phi(\cdot)$, where $\mathcal{I}_d$ is the information matrix of design $d$. The function $\phi(\cdot)$ satisfies three conditions: (i) $\phi(\cdot)$ is convex; (ii) $\phi(c_1 I_{c_0} + c_2 J_{c_0}) \geq \phi(c I_{c_0})$ whenever $c \geq c_1 + c_2$, where $c_1, c_2$ and $c_0$ are scalars and $c_1 I_{c_0} + c_2 J_{c_0}, c I_{c_0}$ are positive definite matrices, and $J_{c_0}$ denotes the $c_0 \times c_0$ matrix with all elements unity, $I_c$ denotes the identity matrix of order $c$; (iii) $\phi(\cdot)$ is permutation invariant. Hence, a universally optimal design is also $D$-, $A$-, and $E$-optimal.

For obtaining the universally optimal designs, we need to show the orthogonality between main effects and two-factor interactions in $S_1$. Hence, a result of Dey and Suen (2002) and Dey et al. (2005) is used in this paper. If all level combinations of the following sets of factors appear equally often in plan $D$: (a) $\{F_u, F_v\}$, $1 \leq u \leq v \leq m$; (b) $\{F_u, F_{i_v}, F_{j_v}\}$, $1 \leq u \leq m$, $1 \leq v \leq k$; (c) $\{F_{i_u}, F_{j_u}, F_{i_v}, F_{j_v}\}$, $1 \leq u \leq v \leq k$, then plan $D$ is universally optimal over $\mathcal{H}$ such that it allows the estimability of the mean, the main effects $F_1, \ldots, F_m$ and the $k$ two-factor interactions $F_{i_1}F_{j_1}, \ldots, F_{i_k}F_{j_k}$, $1 \leq i_u, i_v \leq m$.

However, the problem addressed in this paper is slightly different from the ones considered before. Note that we considered main effects only under the assumption that some two-factor interactions are negligible. Thus, designs containing partially clear main effects may have more columns than the corresponding strength 3 orthogonal arrays with the same runs. By the same technique in Dey and Suen (2002) and Dey et al. (2005), the following theorem on universally optimal plans with partially clear main-effects can be obtained similarly.

**Theorem 2.1.** *A plan $D \in \mathcal{H}$ is universally optimal over $\mathcal{H}$, such that each of $\mathcal{H}$ allows the estimability of the mean and the main effects $F_i$, $i = 1, \ldots, m$, if all level combinations of the following sets of factors appear equally often in $D$:*

(a) $\{F_i, F_j\}$, $1 \leq i \leq j \leq m$;
(b) $\{F_i, F_j, F_k\}$, $1 \leq i \leq m$, $F_jF_k \in S_1$.

For convenience, universally optimal designs for the four cases are still referred to as universally optimal designs of types 1, 2, 3 and 4, respectively.