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Uniform asymptotics for the ruin probabilities of a two-dimensional renewal risk model with dependent claims and risky investments[☆]

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ABSTRACT

Consider a two-dimensional renewal risk model, in which the independent and identically distributed claim-size random vectors follow a common bivariate Farlie–Gumbel–Morgenstern distribution. Assuming that the surplus is invested in a portfolio whose return follows a Lévy process and that the claim-size distribution is heavy-tailed, uniformly asymptotic estimates for two kinds of finite-time ruin probabilities of the two-dimensional risk model are obtained.

1. Introduction

The asymptotic analysis of the ruin probability for one-dimensional renewal risk models has attracted a vast amount of attention due to their practical importance in the past decades; see, e.g., Chen and Ng (2007), Guo and Wang (2013), Kong and Zong (2008), Li (2012), Li et al. (2010), Tang (2005, 2007) and Tang and Yuan (2012) for some recent contributions. However, in many cases insurance companies sell many types of contract, and multi-dimensional risk theory has gained popularity in recent years. When the individual risks are heavy-tailed, the ruin-related problems under multi-dimensional risk models are very complex, even in a two-dimensional case. So far as we known, Chan et al. (2003), Chen et al. (2013b) and Chen et al. (2011) studied ruin probabilities of a two-dimensional risk model without interest force; Huang et al. (2014), Jiang et al. (2015), Li and Yang (2015), Shen and Zhang (2013) and Yang and Li (2014) considered ruin probabilities of a two-dimensional risk model with interest force; Chen et al. (2013a), Gao and Yang (2014), Li et al. (2007) and Zhang and Wang (2012) investigated ruin probabilities of a two-dimensional risk model perturbed by diffusion processes generated by Brownian motions.

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Since it is usual for an insurance company to invest its surplus in a portfolio consisting of risk-free and risky assets, in this paper we are interested in the asymptotics of the finite-time ruin probabilities of a two-dimensional risk model with stochastic returns. Suppose that the price process of the investment portfolio could be modelled by an exponential Lévy process $\{e^{L(t)}; t \geq 0\}$, where $\{L(t); t \geq 0\}$ is a Lévy process, which is commonly used in mathematical finance and actuarial science; see. e.g. Emmer and Klüppelberg (2004), Emmer et al. (2001), Paulsen (2008), Paulsen and Gjessing (1997), among others. Define the Laplace exponent of $\{L(t); t \geq 0\}$ by $\phi(z) = \log \mathsf{E}[e^{-zL(1)}]$, $z \in (-\infty, \infty)$. If $\phi(z)$ is finite, then for $t \geq 0$, $\mathsf{E}[e^{-zL(t)}] = e^{t\phi(z)} < \infty$, and it is easily seen that $\phi(z)$ is convex in z. For the general theory of Lévy process, see Applebaum (2004), Cont and Tankov (2004) and Sato (1999).

Assume that the insurance company has a book of two dependent lines of business sharing the same claim-number process. The total amount of premiums accumulated for the ith class up to time $t \geq 0$ is denoted by $C_i(t)$, which is a non-negative and non-decreasing stochastic process with $C_i(0) = 0$ and $C_i(t) < \infty$ almost surely (a.s.) for every $0 \leq t < \infty$ for $i = 1, 2$. Denote the vector of the total amount of premiums accumulated up to time t by $\vec{C}(t) = (C_1(t), C_2(t))^\top$. Then the surplus process of the insurance company up to time t, denoted by $\vec{U}(\vec{x}, t) = (U_1(x_1, t), U_2(x_2, t))^\top$, satisfies

$$\vec{U}(\vec{x}, t) = e^{L(t)}\left(\vec{x} + \int_{0-}^{t} e^{-L(y)}\vec{C}(\mathrm{d}y) - \int_{0-}^{t} e^{-L(y)}\vec{S}(\mathrm{d}y)\right), \quad t \geq 0, \tag{1.1}$$

where $\vec{x} = (x_1, x_2)^\top$ is the initial surplus vector, $\vec{S}(t) = \sum_{k=1}^{N(t)} \vec{X}_k$ is the vector of the total amount of claims up to time $t \geq 0$, and $\{\vec{X}_k = (X_k^{(1)}, X_k^{(2)})^\top; k \geq 1\}$ is the sequence of claim-size vectors whose common arrival times $\tau_1, \tau_2, \cdots$ constitute a renewal claim-number process $\{N(t); t \geq 0\}$ with finite renewal function $\lambda_t = \mathsf{E}N(t) = \sum_{i=1}^{\infty} \mathsf{P}(\tau_i \leq t)$.

Throughout this paper, $\{\vec{X}_k; k \geq 1\}$ is a sequence of independent and identically distributed (i.i.d.) random vectors with generic vector $(X^{(1)}, X^{(2)})$ whose marginal distribution functions are F_1 and F_2 on $[0, \infty)$, respectively. We assume in this paper that $(X^{(1)}, X^{(2)})$ follows a bivariate Farlie–Gumbel–Morgenstern (FGM) distribution, since in reality an unexpected claim event might produce dependent claims. The joint distribution with marginal distribution functions F_1 and F_2 is given by

$$\Pi(x, y) = F_1(x)F_2(y)(1 + \theta[1 - F_1(x)][1 - F_2(y)]), \quad \theta \in [-1, 1]. \tag{1.2}$$

We further assume that $\{\vec{X}_k; k \geq 1\}$, $\{\vec{C}(t); t \geq 0\}$ and $\{N(t); t \geq 0\}$ are mutually independent.

Consider two types of time of ruin for the above-mentioned two-dimensional renewal risk model, with

$$\tau_{\max}(x_1, x_2) = \inf\{t : \max\{U_1(x_1, t), U_2(x_2, t)\} < 0 \mid U_i(x_i, 0) = x_i,\ i = 1, 2\}$$

being the time when $U_1(x_1, t)$ and $U_2(x_2, t)$ first become negative simultaneously, and

$$\tau_{\min}(x_1, x_2) = \inf\{t : \min\{U_1(x_1, t), U_2(x_2, t)\} < 0 \mid U_i(x_i, 0) = x_i,\ i = 1, 2\}$$

being the time when $U_1(x_1, t)$ or $U_2(x_2, t)$ first becomes negative (see Chan et al., 2003). The corresponding finite-time ruin probabilities are defined as follows:

$$\psi_{\max}(\vec{x}, T) = \mathsf{P}\big(\tau_{\max}(x_1, x_2) \leq T\big) = \mathsf{P}\left(\bigcap_{i=1}^{2}\{U_i(x_i, s) < 0\} \text{ for some } 0 \leq s \leq T\right),$$

and

$$\psi_{\min}(\vec{x}, T) = \mathsf{P}\big(\tau_{\min}(x_1, x_2) \leq T\big) = \mathsf{P}\left(\bigcup_{i=1}^{2}\{U_i(x_i, s) < 0\} \text{ for some } 0 \leq s \leq T\right).$$

In this paper we aim to derive uniform asymptotic estimates for the finite-time ruin probabilities. We present our main results in Section 2 after introducing some necessary preliminaries, and the proofs together with some supporting lemmas are given in Section 3.

2. Notations and main results

In this section, we first recall various definitions used in our subsequent analysis. We shall restrict ourselves to the case of heavy-tailed claim size distributions. A random variable X, with distribution function $F(x) = 1 - \overline{F}(x) > 0$ for $x \in (-\infty, \infty)$ is called heavy-tailed, if $\mathsf{E}(e^{\gamma X}) = \infty$ for all $\gamma > 0$. A distribution F on $[0, \infty)$ belongs to the dominated variation class, denoted by $F \in \mathscr{D}$, if for any $0 < y < 1$, $\limsup_{x\to\infty} \overline{F}(xy)/\overline{F}(x) < \infty$. Another useful heavy-tailed class is the class $\mathscr{L}$ of all distribution functions with long tail, characterized by the relation $\lim_{x\to\infty} \frac{\overline{F}(x-y)}{\overline{F}(x)} = 1$ for any $y > 0$. In this paper we are particularly interested in the class $\mathscr{L} \cap \mathscr{D}$, which is a very large heavy-tailed subclass and is rich enough to contain many useful heavy-tailed distributions in modelling risk variables. One of the most important subclasses of $\mathscr{L} \cap \mathscr{D}$ is the class of distributions with regularly-varying tails. By definition, a distribution F is said to have a regularly-varying tail if $\lim_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = y^{-\alpha},\ y > 0$ holds for some $\alpha \geq 0$. In this case, we write $F \in \mathscr{R}_{-\alpha}$.