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Anticipative backward stochastic differential equations driven by fractional Brownian motion

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ABSTRACT

We study the anticipative backward stochastic differential equations (BSDEs, for short) driven by fractional Brownian motion with Hurst parameter H greater than 1/2. The stochastic integral used throughout the paper is the divergence operator type integral. We obtain the existence and uniqueness theorem to these equations under the Lipschitz condition. A comparison theorem for this type of anticipative BSDEs is also established.

1. Introduction

Fractional Brownian motion (fBm, for short) with Hurst parameter $H \in (0, 1)$ is a zero mean Gaussian process $B^H = \{B^H_t, t \geq 0\}$ whose covariance is given by

$$\mathbb{E}(B^H_t B^H_s) = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H}).$$

For $H = \frac{1}{2}$, the process B^H is a classical Brownian motion. In the case $H > \frac{1}{2}$, the process B^H exhibits long range dependence. These properties make this process a useful driving noise in models arising in finance, physics, telecommunication networks and other fields. However, since B^H with $H > \frac{1}{2}$ is not a semimartingale, we cannot use the classical theory of stochastic calculus to define the fractional stochastic integral. Essentially, two different types of integrals with respect to fBm have been defined and developed. The first one is the pathwise Riemann–Stieltjes integral which exists if the integrand has a continuous paths of order $\alpha > 1-H$ (see Young, 1936). This type of integrals has the properties of Stratonovich integral, which can lead to difficulties in applications. The second one, introduced by Decreusefond and Üstünel (1999), is the divergence operator (Skorohod integral), defined as the adjoint of the derivative operator in the framework of the Malliavin calculus. Since this stochastic integral satisfies the zero mean property and it can be expressed as the limit of Riemann sums by using Wick products, it was later developed by many authors. We refer to the works of Biagini et al. (2008) and Nualart (2006).

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Backward stochastic differential equations (BSDEs in short) driven by a Brownian motion were introduced by Bismut (1973) for the linear case and by Pardoux and Peng (1990) in the general case. Since then, these pioneer works are extensively used in many fields like mathematical finance (El Karoui et al., 1997), stochastic optimal control and stochastic games (Hamadène and Lepeltier, 1995). Recently, Peng and Yang (2009) introduced a new type of BSDEs, called anticipative BSDEs, which can be regarded as a new duality type of stochastic differential delay equations. BSDEs driven by fBm were firstly studied by Hu (2005) and Hu and Peng (2009), where they obtained the existence and uniqueness of the solution. Then Hu et al. (2012) established a comparison result for fractional BSDEs. Some other recent developments of fractional BSDEs can be found in Bender (2014), Borkowska (2013) and Jing (2012), etc.

Motivated by the above works, the purpose of this paper is to study the anticipative BSDEs driven by fBm with Hurst parameter $H > \frac{1}{2}$. Under the Lipschitz condition, we prove this type of equations admits a unique solution. As a fundamental tool, the comparison theorem plays an important role in the theory and applications of BSDEs. We also establish a comparison theorem for this class of anticipative BSDEs.

This paper is organized as follows. In Section 2, we provide some basic results on fractional Brownian motions. Section 3 contains the definition of anticipative BSDEs with respect to the fBm. The existence and uniqueness result is proved here. We give a comparison theorem for the solutions of anticipative BSDEs in Section 4.

2. Fractional calculus

Let $(\Omega, \mathcal{F}, P, \mathcal{F}_t,\ t \geq 0)$ be a complete stochastic basis such that $\mathcal{F}_0$ contains all P-null elements of $\mathcal{F}$ and suppose that the filtration is generated by a fractional Brownian motion $B^H = \{B^H_t, t \geq 0\}$. We assume $H > \frac{1}{2}$ throughout this paper. Denote $\phi(x) = H(2H-1)|x|^{2H-2},\ x \in \mathbb{R}$. Let ξ and η be two continuous functions on $[0, T]$. We define

$$\langle \xi, \eta \rangle_t = \int_0^t \int_0^t \phi(u-v)\xi_u \eta_v du dv,$$

and $\|\xi\|_t^2 = \langle \xi, \xi \rangle_t$. Note that, for any $t \in [0, T]$, $\langle \xi, \eta \rangle_t$ is a Hilbert scalar product. Let $\mathcal{H}$ be the completion of the continuous functions under this Hilbert norm. The elements of $\mathcal{H}$ may be distributions.

We denote by $\mathcal{P}_T$ the set of all polynomials of fractional Brownian motion in $[0, T]$, i.e., it contains all elements of the form

$$F(\omega) = f\left(\int_0^T \xi_1(t) dB^H_t, \ldots, \int_0^T \xi_n(t) dB^H_t\right),$$

where f is a polynomial function of n variables. The Malliavin derivative operator D^H_s of an element $F \in \mathcal{P}_T$ is defined as follows:

$$D^H_s F = \sum_{i=1}^n \frac{\partial f}{\partial x_i}\left(\int_0^T \xi_1(t) dB^H_t, \ldots, \int_0^T \xi_n(t) dB^H_t\right)\xi_i(s), \quad s \in [0, T].$$

Since the divergence operator $D^H : L^2(\Omega, \mathcal{F}, P) \to (\Omega, \mathcal{F}, \mathcal{H})$ is closable, we can consider the space $\mathbb{D}^{1,2}$ be the completion of $\mathcal{P}_T$ with the norm

$$\|F\|_{1,2}^2 = \mathbb{E}|F|^2 + \mathbb{E}\|D^H_s F\|_T^2.$$

We also introduce another derivative

$$\mathbb{D}^H_t F = \int_0^T \phi(t-s) D^H_s F ds.$$

Denote by $\mathbb{L}^{1,2}_H$ the space of all stochastic processes $F : (\Omega, \mathcal{F}, P) \to \mathcal{H}$ such that

$$\mathbb{E}\left(\|F\|_T^2 + \int_0^T \int_0^T |\mathbb{D}^H_s F_t|^2 ds dt\right) < \infty.$$

The following results are well know now (see Duncan et al., 2000, Hu, 2005, Hu and Øksendal, 2003).

Proposition 2.1. *Let $F \in \mathbb{L}^{1,2}_H$, then the Itô–Skorohod type stochastic integral $\int_0^T F_s dB^H_s$ exists in $L^2(\Omega, \mathcal{F}, P)$. Moreover, we have*

$$\mathbb{E}\left(\int_0^T F_s dB^H_s\right) = 0,$$

and

$$\mathbb{E}\left(\int_0^T F_s dB^H_s\right)^2 = \mathbb{E}\left(\|F\|_T^2 + \int_0^T \int_0^T \mathbb{D}^H_s F_t \mathbb{D}^H_t F_s ds dt\right).$$