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On maximizing expected discounted taxation in a risk process with interest ☆

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ABSTRACT

In risk theory, the problem of maximizing the expected cumulated discounted loss-carry-forward tax payments until ruin is a widely discussed topic since the taxation system was proposed by Albrecher and Hipp (2007). In the present paper, we discuss this maximization problem in the Cramér–Lundberg risk model including a constant force of interest. The optimal taxation return function is identified as the classical solution of the associated Hamilton–Jacobi–Bellman equation and the optimal taxation strategy in this risk model with interest is derived, which is of band type. Finally, an example is constructed for exponential claim sizes, in which closed-form expression for the optimal taxation return function is given.

1. Introduction

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a filtered probability space on which all random processes and variables introduced in the following are defined. We assume that, in the case of no loss-carry-forward tax (see Albrecher and Hipp, 2007, where the so-called loss-carry-forward tax was first introduced), the surplus process $\{U_t; t \geq 0\}$ with $U_0 = x$ of an insurance portfolio evolves according to

$$dU_t = (rU_t + c)dt - d\left(\sum_{k=1}^{N(t)} Y_k\right), \tag{1.1}$$

where $r > 0$ is the constant force of interest on the free surplus (see for instance Paulsen (1993)), $c > 0$ the premium rate, $\{N(t),\ t \geq 0\}$ a Poisson process (with jump intensity $\lambda > 0$) denoting the number of claims up to time t, and $\{Y_n,\ n \geq 1\}$ (representing the amounts of claims and independent of $\{N(t),\ t \geq 0\}$) an i.i.d. sequence of positive random variables with a

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continuous distribution function $F(y)$. For the sake of simplicity, let $p(y) := F'(y)$ be the individual claim amount probability density function.

Wei (2009) extended risk model (1.1) by incorporating a surplus-dependent loss-carried-forward tax structure. Namely, the author considered the following stochastic model $\{U_t^\pi;\ t \geq 0\}$ characterized as

$$dU_t^\pi = (rU_t^\pi + c)(1 - \gamma^\pi(U_t^\pi)I_{\{U_t^\pi = S_t^\pi\}})dt - d\left(\sum_{k=1}^{N(t)} Y_k\right), \tag{1.2}$$

where $S_t^\pi := \max_{0\leq s\leq t} U_s^\pi$ is the running supremum process of $\{U_t^\pi;\ t \geq 0\}$, and $\{\gamma^\pi(U_t^\pi);\ t \geq 0\}$ with $\gamma^\pi(y) \in [0, 1)$ for all $y \geq 0$ is a loss-carried-forward taxation strategy. An explicit asymptotic formula for the ruin probability of (1.2) for subexponential claims was obtained.

Following Wei (2009), Wang et al. (2010) studied the same model. For a fixed constant tax rate, the authors addressed the problem of choosing an optimal surplus threshold for starting taxation to maximize the expected discounted tax payments. A sufficient condition for the uniqueness of the optimal taxation level was thus derived.

Motivated by Wei (2009) and Wang et al. (2010), the present paper is concerned with the taxation maximization problem in risk model (1.2). Specifically, we shall find the optimal taxation return function and the optimal taxation strategy which maximizes the expected cumulated discounted tax payments until ruin. It is proved that the optimal taxation strategy is of band type.

It needs to be mentioned that when our model and tax structure are specified to those of Wang et al. (2010), then our results are the same as the corresponding results of Wang et al. (2010), see Remark 4.1. It also needs to be mentioned that, under the risk model (1.2) associated with (1.1) which is not a Lévy setup, more direct arguments will be employed. For more related references on the loss-carry-forward tax problem, the reader may consult the following publications and references therein, Wang and Hu (2012), Wang et al. (2011), Ming et al. (2010), Albrecher et al. (2008), Albrecher et al. (2007), and Kyprianou and Zhou (2009), etc.

Mathematically, a loss-carried-forward taxation strategy described by a one-dimensional stochastic process $\{\gamma^\pi(U_t^\pi);\ t \geq 0\}$ (or, with a little abuse of notation, simply denoted by π) is called admissible if the map $\gamma^\pi : [0, \infty) \to [\alpha, \beta]$ with $0 \leq \alpha \leq \beta < 1$ is measurable. By Π we denote the set of all admissible taxation strategies. For a given admissible strategy $\pi \in \Pi$, we define the performance function V_π by

$$V_\pi(x) = \mathbb{E}_x \int_0^{\tau_\pi} e^{-\delta t}(rU_t^\pi + c)\gamma^\pi(U_t^\pi)I_{\{U_t^\pi = S_t^\pi\}}dt, \tag{1.3}$$

where $\tau_\pi = \inf\{t \geq 0; U_t^\pi < 0\}$ is the time of ruin, $\delta > r$ a discount factor and $\mathbb{E}_x$ the expectation corresponding to the law of $\{U_t;\ t \geq 0\}$ such that $U_0 = x$. The objective is to find the optimal taxation return function,

$$V(x) = \sup_{\pi\in\Pi} V_\pi(x) \tag{1.4}$$

and to find an optimal taxation strategy π^* that satisfies $V(x) = V_{\pi^*}(x)$ for all x.

Let $h : [0, \infty) \to [0, \infty)$ be the unique (up to multiplication by a constant) solution of the integro-differential equation

$$(rx + c)h'(x) - (\lambda + \delta)h(x) + \lambda \int_0^x h(x - y)dF(y) = 0. \tag{1.5}$$

Throughout this paper, we assume that the positive safety loading condition $c > \lambda\mu$ holds, where μ denotes the mean claim size.

The rest of the paper is organized as follows. In Section 2 we assume that the optimal taxation return function is once continuously differentiable and find that under this assumption it satisfies a Hamilton–Jacobi–Bellman (HJB) equation. It is proved, in Section 3, that a solution to the HJB equation coincides with the optimal taxation return function. In Section 4, solutions to the HJB equation are constructed and the optimal taxation strategy is also found. Finally, in Section 5, closed-form expression of the optimal taxation return function is given when the claim sizes are exponentially distributed.

2. The Hamilton–Jacobi–Bellman equation

Define

$$\tau_b = \inf\{t \geq 0;\ U_t > b\} \quad \text{and} \quad \tau_b^\pi = \inf\{t \geq 0;\ U_t^\pi > b\}, \tag{2.1}$$

with the convention that $\inf \phi = \infty$. In addition, let

$$B(x, b) = \mathbb{E}_x[e^{-\delta\tau_b} I_{\{\tau_b<\tau\}}] \quad \text{and} \quad B_\pi(x, b) = \mathbb{E}_x[e^{-\delta\tau_b^\pi} I_{\{\tau_b^\pi<\tau_\pi\}}], \tag{2.2}$$

where τ is time of ruin of the risk process $\{U_t;\ t \geq 0\}$ defined as $\tau = \inf\{t \geq 0;\ U_t < 0\}$. It is known that $B(x, b) = \frac{h(x)}{h(b)}$.