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On the law of large numbers for discrete Fourier transform

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ABSTRACT

We shall establish the law of large numbers for the discrete Fourier transform of random variables with finite first moment under condition $P(|X_n| > x) \leq P(|X_1| > x)$ for all $x \geq 0$; for $1 < p < 2$, we establish the Marcinkiewicz–Zygmund type rate of convergence for the discrete Fourier transform of random variables with finite *pth* moment under condition $\frac{1}{n}\sum_{k=1}^{n} P(|X_k| > x) \leq MP(|X_1| > x)$ for all $x \geq 0$ and some positive constant M.
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1. Introduction

The law of large numbers is valid for pairwise independent identically distributed random variables, result due to Etemadi (1981). This is a surprising result since a sequence of pairwise independent identically distributed random variables may not be ergodic. A way to look into the speed of convergence of this result when the variables have finite moments of order r, $1 < r < 2$, is provided by Baum and Katz (1965) in the i.i.d. case and by Stoica (2011) in the martingale difference case. By carefully examining the proof in Stoica (2011), we notice that the proof can be adapted to centered pairwise independent random variables and we can formulate the following result.

Proposition 1.1. *Let $(X_n)_{n\geq 1}$ be a pairwise independent sequence of random variables with the same distribution.*

(a) *Assume $X_n \in L^1$. Then*

$$\frac{S_n}{n} \to EX_1, \quad P\text{-a.s. as } n \to \infty,$$

where $S_n = \sum_{k=1}^{n} X_k$.

(b) *Assume $X_n \in L^p$, $1 < p < 2$. Then for all $1 \leq r \leq p$ and $\epsilon > 0$,*

$$\sum_{n=1}^{\infty} n^{p/r-2} P\left(|S_n| > \epsilon n^{1/r}\right) < \infty.$$

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We note that, by Korchevsky (2015), $\frac{S_n - ES_n}{n^{1/p}} \to 0$, P-a.s. as $n \to \infty$, also holds.

The goal of our note is to study the law of large numbers for the discrete Fourier transform of a sequence of random variables with finite *pth* moment for $p = 1$ and its Marcinkiewicz–Zygmund type rate of convergence for $1 < p < 2$ under weaker conditions than identical distribution as described in the abstract and to show that, from some point of view, the variables have similar properties as pairwise independent random variables with the same distribution.

2. Main results

Let $(X_n)_{n\geq 1}$ denote a sequence of real-valued random variables on a probability space $(\Omega, \mathcal{F}, P)$. No dependence between the variables is assumed. For $-\pi \leq t < \pi$, define the discrete Fourier transform

$$S_n(t) = \sum_{k=1}^{n} e^{ikt} X_k. \tag{1}$$

We shall establish an analogue of the result by Baum and Katz (1965) for the discrete Fourier transform.

Theorem 2.1. *Assume*

$$P(|X_n| > x) \leq P(|X_1| > x), \quad \text{for all } x \geq 0,\ n \geq 1. \tag{UD}$$

If $E|X_1| < \infty$*, then for almost all* $t \in [-\pi, \pi)$,

$$\lim_{n\to\infty} \frac{S_n(t)}{n} = 0, \quad P - a.s.$$

The following theorem describes the rate of convergence in the strong law of large numbers.

Theorem 2.2. *Assume*

$$\frac{1}{n}\sum_{k=1}^{n} P(|X_k| > x) \leq MP(|X_1| > x), \quad \text{for all } x \geq 0,\ n \geq 1. \tag{MD}$$

Let $1 < p < 2$, $1 \leq r \leq p$*. If* $E|X_1|^p < \infty$*, then for every* $\epsilon > 0$ *and for almost all* $t \in [-\pi, \pi)$,

$$\sum_{n=1}^{\infty} n^{p/r-2} P[\max_{1\leq k\leq n} |S_k(t)| > \epsilon n^{1/r}] < \infty. \tag{2}$$

Remark 2.3. In fact, Gut (1992) gave an example to show that condition (MD) is strictly weaker than condition (UD). But in Theorem 2.1, it is an open question whether condition (UD) can be weakened to condition (MD).

Remark 2.4. Differently from the i.i.d. case, the reciprocal of Theorem 2.2 is no longer true. That is, (2) does not imply X_1 have finite *pth* moment.

Corollary 2.5. *Under the assumption of Theorem 2.2, for almost all* $t \in [\pi, \pi)$,

$$\lim_{n\to\infty} \frac{S_n(t)}{n^{1/p}} = 0, \quad P\text{-}a.s.$$

Remark 2.6. Both theorems hold if the variables have the same distribution.

3. Proofs

Throughout this whole paper, $C > 0$ denotes a generic constant which may take different values from line to line.

In order to prove our main theorems, we shall first establish two preparatory lemmas. We begin by a truncation argument.

Lemma 3.1. *Assume* $(X_n)_{n\geq 1}$ *satisfies condition (UD) and* $E|X_1| < \infty$*. Let* $Y_k = X_k I\{|X_k| \leq k\}$ *and* $S_n^*(t) = \sum_{k=1}^{n} e^{ikt} Y_k$*. Then for all* t *in* $[-\pi, \pi)$,

$$\lim_{n\to\infty} \left| \frac{1}{n} S_n(t) - \frac{1}{n} S_n^*(t) \right| = 0, \quad P\text{-}a.s.$$

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