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A Burton–Miller formulation of the boundary element method for baffle problems in acoustics and the BEM/FEM coupling

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ABSTRACT

Baffle problems, i.e. radiation problems from objects mounted behind a hole of an infinite hard reflecting wall, can be simulated as a multi domain problem consisting of a finite interior domain around the object, and two infinite half spaces in front and behind the baffle plane. A formulation of such problems is presented in the context of the Burton–Miller boundary element method. Additionally, the coupling of the acoustic boundary element method and the structural finite element method in the context of the Burton–Miller-formulation of the baffle problem is discussed.

1. Introduction

It is well-known, that for certain critical frequencies the integral representation formula for an exterior Helmholtz problem does not have a unique solution. The CHIEF-point method presented by Schenck [1] is widely used to guarantee a unique solution also at these irregular frequencies. However, particularly at higher frequencies, it is difficult to select appropriate CHIEF-points, because they must not be located near the nodal surfaces of an associated eigenvalue problem which are unknown (see also [2–4]). Moreover, the large condition number of the coefficient matrix, which is caused by improperly selected CHIEF-points, makes the application of iterative solvers almost impossible. However, for large scale problems, especially when fast numerical methods such as the fast multipole method [3,5–11] are used, iterative solvers are preferred or even inevitable, thus a high condition number caused by badly selected CHIEF points should be avoided. The Burton–Miller method [12] is free from the above difficulties. In [13] an algorithm for a symmetric Galerkin-type boundary element method (BEM) with CHIEF-points was formulated. Based on the equations derived in that work, we formulate equations for a Burton–Miller BEM for acoustic radiation and scattering in a similar manner.

The baffle problem, i.e. the acoustic radiation problem including an apparatus mounted behind a hole of a large, hard reflecting wall, is of practical importance for developing electrical devices such as high frequency loudspeakers. To simulate the reflections from the baffle, the baffle can be discretized directly, but this approach is not efficient, because the baffle is generally seen as being infinitely large compared to the apparatus. Alternatively, in our approach an interface that encloses the considered apparatus is chosen, so that the whole space is subdivided into three domains: a finite one around the apparatus and the hole in the baffle, and two infinite ones in front and behind the baffle, respectively (see Fig. 1). The three-domain approach was chosen to correctly model situations, when the apparatus is mounted behind a hole in the baffle. The apparatus may lie partly in front of the baffle and partly behind it. The advantage of our approach is that only the interface and the surface of the apparatus need to discretized, but not the baffle itself. Thus the dimension of the problem can be reduced drastically compared to the direct method. Multi domain approaches were already used for coupled interior/exterior acoustic problems (see for example [14]) or for modeling seismic wave propagation [15].

In Section 2, we derive the boundary integral equations (BIE) for baffle problems in the context of the Burton–Miller BEM and derive the linear system of equations. Naturally, the boundary integral equations for problems without baffle can be seen as a special case of that for the baffle problem. In Section 3, the coupling between the Burton–Miller BEM for the baffle problem and the structural finite element method (FEM) will be treated using the similar approach as in [16,17]. In Section 4, results for three examples are shown. In the first example, a baffle problem is solved using the direct method (i.e. direct discretization of a large part of the baffle) and the indirect method (i.e. using a multi domain model without discretization of the baffle itself). The second example shows the results for the sound radiation from a high frequency loudspeaker mounted on a baffle. The third example is the simulation of sound

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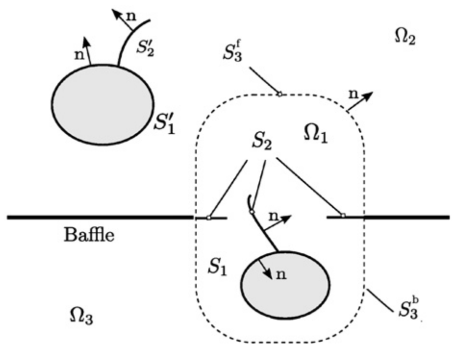

Fig. 1. Schematic illustration of a three domain baffle problem.

reflection from a closed surface discretized with surface elements. To show that the Burton–Miller method can enhance the numerical stability and accelerate the convergence of the iterative solver for exterior and interior problems, an exterior problem as well as an interior problem are treated.

The linear system of equations describing the system is solved with an iterative solver. Because of the Burton Miller approach the system is not symmetric anymore, as iterative solver we use a conjugate gradient squared method (CGS) [18]. Although it was shown in [19,20] that the BiCGStab2 solver has some robustness advantage over the CGS method, we found in our experience, that the CGS solver with an incomplete LU-decomposition as preconditioner has a good balance between stability and efficiency, and is therefore used in our code. For further discussion of iterative solvers for BEM problems please refer to [19,21,22].

2. The boundary integral equations for acoustic problems under consideration of a baffle

The basic equation for acoustic problems in the frequency domain is the Helmholtz equation

$$\nabla^2\phi + k^2\phi = 0, \tag{1}$$

with ϕ denoting the velocity potential and $k=\omega/c$ denoting the wave number (ω and c are the angular frequency and the sound speed, respectively). The sound pressure is related to ϕ by $p=\mathrm{i}\omega\rho\phi$, with ρ being the density of the medium and $\mathrm{i}^2=-1$. Here it is assumed, that the time factor is $e^{-\mathrm{i}\omega t}$, otherwise $p=-\mathrm{i}\omega\rho\phi$. The baffle plane is assumed to be hard reflecting, the radiating/reflecting objects are assumed to have sound-absorbing properties modeled by an admittance boundary condition, i.e.

$$v(\boldsymbol{x}) = \frac{\partial\phi(\boldsymbol{x})}{\partial\mathbf{n}} = v_{\mathrm{rad}}(\boldsymbol{x}) - \tau A(\boldsymbol{x})\phi(\boldsymbol{x}), \quad \boldsymbol{x}\in S, \tag{2}$$

where $A(\boldsymbol{x})=\mathrm{i}\omega\rho a(\boldsymbol{x})$ is given by the admittance $a(\boldsymbol{x})$ at a point $\boldsymbol{x}$ on the boundary S, $\mathbf{n}$ is the normal vector to the surface of the structure at point $\boldsymbol{x}$, and v_{rad} is given by the (possible) sound radiation of the object modeled by a velocity boundary condition. The parameter τ depends on the direction of the normal vector to the surface (see Fig. 1). If the normal vector points to the interior of the radiating/reflecting object, $\tau := -1$, otherwise $\tau := 1$. In Fig. 1, a three-domain baffle problem is depicted. The full domain is subdivided by an artificial interface $S_3 = S_3^f \cup S_3^b$ (interface part in front of the baffle and the interface part behind the baffle) into the sub-domains Ω_1, Ω_2 and Ω_3. Ω_1 is a bounded domain where the radiating apparatus and the hole in the baffle are located; Ω_2 and Ω_3 are two semi infinite domains in front and behind the baffle. S_1 and S_1' denote the surfaces of radiating or reflecting objects in Ω_1 and Ω_2, respectively. S_2 and S_2' are thin-walled structural elements (middle face elements). The interface S_3 will be discretized with special elements to guarantee the continuity of sound pressure and normal particle velocity across the interface. A part of the baffle can be in Ω_1, this part has to be modeled by middle face elements, i.e. elements in S_2. In Ω_2 and Ω_3, the baffle is modeled by considering both domains as hard reflecting infinite half spaces and using the appropriate fundamental solution for the Helmholtz equation in such spaces. Thus the baffle does not have to be discretize in these half spaces. We assume, that Ω_2 may contain reflecting objects, whereas in Ω_3 no objects may be given. Please note, that this restriction is only made, because in most real life applications (like for example high frequency loudspeakers) there are no objects located in Ω_3. The problem in the domain Ω_1 is defined as an interior problem, and the normal vector to the surface S_1 must point to the interior of the object (that is an convention made in [13] to ensure that the direction of the normal vectors to the interface S_3 match when the domains are coupled); please note that Ω_1 is the domain between the artificial boundary S_3 and the radiating object. In the domain Ω_2, the normal vector to S_1' must point to the exterior of the radiating/reflecting object, because the problem is defined as an exterior problem there. The direction of the normal vector to thin elements is arbitrary, and it is used to define the positive and negative sides of S_2 and S_2'. If a part of the apparatus behind the baffle closes the baffle (for example a cabinet behind the baffle, see Fig. 2), and the elements of this closure can be modeled by surface elements, these elements can be used to separate Ω_1 from the halfspace behind the baffle, i.e. the closure can be used instead of S_3^b. Since everything behind the baffle and the closure is not relevant for the calculations anymore, the three-domain model can be reduced to a two domain model.

In [13] the boundary integral equations of a Galerkin-BEM-formulation for acoustic scattering for surface and thin-walled structural elements was given. CHIEF-points were used to remove the singularities of the system of equations at irregular frequencies. The Burton–Miller formulation and the baffle problem were not considered yet. Based on these equations, we derive a formulation for a baffle problem in combination with the Burton–Miller approach. First, we will derive the equations for points in the closed domain, then for points in the half spaces. We then apply the Burton Miller approach to these equations. Finally, we will derive equations for points at the interface between the three domains.

2.1. BIE for the closed domain Ω_1

We first take a look at Ω_1, i.e. the region between the radiating object and the interface S_3, where the problem is defined as an interior problem. Please note, that in the subsystem, points on the interface boundary S_3 can be treated the same way as points on regular surface elements S_1. The boundary integral equations are given by

$$-\alpha\phi(\boldsymbol{x}) + \int_{S_1} (G(\boldsymbol{x},\boldsymbol{y})A(\boldsymbol{y}) - H(\boldsymbol{x},\boldsymbol{y}))\phi(\boldsymbol{y})\, dS_y$$
$$+ \int_{S_2} ((G(\boldsymbol{x},\boldsymbol{y})\tilde{A}(\boldsymbol{y}) + H(\boldsymbol{x},\boldsymbol{y}))\tilde{\phi}(\boldsymbol{y}) + G(\boldsymbol{x},\boldsymbol{y})\overline{A}(\boldsymbol{y})\overline{\phi}(\boldsymbol{y}))\, dS_y$$

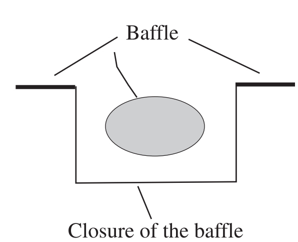

Fig. 2. Schematic drawing of a two-domain baffle. The hole in the baffle is closed, thus contributions from Ω_3 are not relevant for calculations. The closure forms the boundary of Ω_1 behind the baffle and has to be modeled with surface elements in S_1.