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A regular variational boundary model for free vibrations of magneto-electro-elastic structures

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ABSTRACT

In this paper a regular variational boundary element formulation for dynamic analysis of two-dimensional magneto-electro-elastic domains is presented. The method is based on a hybrid variational principle expressed in terms of generalized magneto-electro-elastic variables. The domain variables are approximated by using a superposition of weighted regular fundamental solutions of the static magneto-electro-elastic problem, whereas the boundary variables are expressed in terms of nodal values. The variational principle coupled with the proposed discretization scheme leads to the calculation of frequency-independent and symmetric generalized stiffness and mass matrices. The generalized stiffness matrix is computed in terms of boundary integrals of regular kernels only. On the other hand, to achieve meaningful computational advantages, the domain integral defining the generalized mass matrix is reduced to the boundary through the use of the dual reciprocity method, although this implies the loss of symmetry. A purely boundary model is then obtained for the computation of the structural operators. The model can be directly used into standard assembly procedures for the analysis of non-homogeneous and layered structures. Additionally, the proposed approach presents some features that place it in the framework of the weak form meshless methods. Indeed, only a set of scattered points is actually needed for the variable interpolation, while a global background boundary mesh is only used for the integration of the influence coefficients. The results obtained show good agreement with those available in the literature proving the effectiveness of the proposed approach.

1. Introduction

The use of magneto-electro-elastic materials has recently emerged in the field of smart structural devices such as sensors, actuators and transducers. These media convert energy into three different forms: magnetic, electric and mechanic. This peculiar feature arises as magneto-electro-elastic media are particulate or laminate composite materials having among their constituents piezoelectric and piezomagnetic phases. Magneto-electric coupling is the by-product property of magneto–mechanic and electro–mechanic coupling and this characteristic seems to make these multi-field composites potentially superior to other materials for application in smart structures [1]. The properties of these materials allow employing electrically or magnetically induced strains to control the mechanical behaviour of a structure and, on the other hand, the use of strain-induced electric and magnetic signals as a feedback driver in control systems. At its more evolved application this approach leads to the concept of highly integrated intelligent structures which are capable of reacting to external stimula by means of a distributed network of sensors and actuators [2–4]. This goal can be achieved by layers of magneto-electro-elastic material being either attached to or embedded in the host structure to be controlled and actuated by a system of suitably arranged electrodes and poles. Among other technological applications, great interest is devoted to structural vibrations control through the use of active or passive damping systems based on magneto-electro-elastic materials as just proposed with piezoelectrics [5–8] and piezomagnetics [9–11].

The effective design of magneto-electro-elastic devices relies on the capability of correctly modelling the system's response taking into account the mutual effects between mechanical, electric and magnetic fields. Analytical solutions to problems concerning magneto-electro-elastic solids are rare due to the complexity of the governing equations. Recently, some formulations have been developed for the dynamic analysis of single-layer and laminated electro-elastic [12–18] and magneto-electro-elastic solids [19–25]. These models are generally based on energy principles and are classified on the basis of the assumptions made to approximate the electromechanical variables in the thickness direction of the laminate. They are analytically solved for simple cases, whereas the extensive use of the finite element method allows the treatment of more general problems.

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Besides the finite element techniques, the boundary element method (BEM) has been gaining attention over the last few decades as a powerful and reliable tool for the accurate calculation of the solution to several problems in many different areas [26]. It has been recently extended to the study of piezoelectric [27–29] and magneto-electro-elastic [30–33] problems. When dealing with anisotropic, piezoelectric or magneto-electro-elastic dynamics by the boundary element method, the main drawback is the lack of the corresponding fundamental solutions, which generally leads to rather expensive numerical integration schemes. This issue is addressed by several authors through the use of the fundamental solutions of the associated static problem and by considering the inertia terms as body forces. The domain integrals obtained from this procedure are generally transformed to boundary integrals through the use of appropriate techniques such as the dual reciprocity method [34,35]. Indeed, this approach has successfully been applied to the free vibrations analysis of anisotropic [36–38] and piezoelectric [39,40] structures. Nonetheless, in the standard BEM dynamic models the properties of symmetry and definiteness of the continuum are lost and the discrete structural operators are neither symmetric nor definite. The loss of these fundamental properties of the continuum makes BEM models less efficient for dynamics. To overcome this issue, variational BEM formulations have been developed for both elastostatics and elastodynamics [41–46]. In this context, the displacement boundary method (DBM) proposed by Davì and Milazzo for the solution of 2D free and forced vibrations of isotropic domains [47,48] and for the lateral vibration analysis of isotropic and anisotropic plates [49,50], represents a variational formulation devised to preserve the fundamental properties of the structural matrices. Moreover, as pointed out in Ref. [50], this approach presents some features which allow classifying it as a weak form meshless method [51]. Indeed, only a set of scattered points is actually needed for the variable interpolation, while a background boundary mesh is only used for the integration of the influence coefficients. The present paper extends this variational formulation to the magneto-electro-elastic problem. In addition an improved discretization scheme to obtain the corresponding numerical model is presented. The model is also applied to the analysis of multidomain configurations for the study of multilayer structures. To assess the accuracy and the effectiveness of the proposed approach, some applications are presented and the results are compared with those available in the literature.

2. Governing equations

Let Ω denote a two-dimensional magneto-electro-elastic body lying in the x_1x_2 plane and bounded by the contour line $\partial\Omega$. It is assumed that the magneto-electro-mechanical response does not vary along the x_3 direction. Moreover, despite the non-linear behaviour of the magnetostrictive phase, it is assumed that the material behaves linearly: in fact, most of the common magnetostrictive materials exhibit moderate linearity when undergoing small excitations. The dynamic elastic state of the body is described using the displacement field $\mathbf{u}^T=[u_1 \quad u_2]$, the strain vector $\boldsymbol{\gamma}^T=[\gamma_{11} \quad \gamma_{22} \quad \gamma_{12}]$ and the stress vector $\boldsymbol{\sigma}^T=[\sigma_{11} \quad \sigma_{22} \quad \sigma_{12}]$. In magneto-electro-elastic materials the elastic waves propagate at a speed several orders of magnitude lower than the speed of the electromagnetic waves. This implies that the electromagnetic field can be treated as quasi-static and dynamical changes in the elastic field result in instantaneous change in the electric and magnetic fields. Therefore, the electric state is described in terms of the electric potential function φ, the electric field vector $\mathbf{E}^T=[E_1 \quad E_2]$ and the electric displacements vector $\mathcal{D}^T=[D_1 \quad D_2]$. To model the magnetic state, it is assumed that no external current density is present in the domain and the scalar magnetic potential ψ is therefore chosen as the magnetic field principal variable. The associated magnetic field $\mathbf{H}^T=[H_1 \quad H_2]$ and magnetic induction $\mathbf{B}^T=[B_1 \quad B_2]$ are also introduced.

Extending the Barnett and Lothe's formalism [52] for piezoelectrics to magneto-electro-elasticity, a compact matrix notation for the problem governing equations can be achieved. Let us define a set of generalized quantities, namely generalized displacements $\mathbf{U}=[u_1 \quad u_2 \quad \varphi \quad \psi]^T$, generalized stresses $\boldsymbol{\Sigma}^T=[\boldsymbol{\sigma}^T \quad \mathbf{D}^T \quad \mathbf{B}^T]$, generalized strains $\boldsymbol{\Gamma}^T=[\boldsymbol{\gamma}^T {-\mathbf{E}^T} \quad {-\mathbf{H}^T}]$ and generalized body forces $\mathbf{F}=[f_1 \quad f_2 \quad -\omega \quad 0]^T$, where f_i are the components of the mechanical body forces and ω is the electric charge density. The generalized compatibility relationships (i.e. the strain–displacement, electric gradient and magnetic gradient relationships), the generalized equilibrium equations (i.e. the equilibrium equations and Gauss' laws for electrostatic and magnetostatics) and the magneto-electro-elastic constitutive law are written as

$$\boldsymbol{\Gamma}=\mathcal{D}\mathbf{U} \quad \text{in } \Omega \tag{1}$$

$$\mathcal{D}^T\boldsymbol{\Sigma}+\mathbf{F}=0 \quad \text{in } \Omega \tag{2}$$

$$\boldsymbol{\Sigma}=\mathbf{R}\boldsymbol{\Gamma} \quad \text{in } \Omega \tag{3}$$

where $\mathbf{R}$ is the generalized stiffness matrix and $\mathcal{D}$ the generalized compatibility operator defined as follows:

$$\mathbf{R}=\begin{bmatrix} \mathbf{C} & \mathbf{e}^T & \mathbf{d}^T \\ \mathbf{e} & -\boldsymbol{\varepsilon} & -\mathbf{g}^T \\ \mathbf{d} & -\mathbf{g} & -\boldsymbol{\mu} \end{bmatrix} \tag{4}$$

$$\mathcal{D}=\begin{bmatrix} \partial/\partial x_1 & 0 & \partial/\partial x_2 & 0 & 0 & 0 & 0 \\ 0 & \partial/\partial x_2 & \partial/\partial x_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \partial/\partial x_1 & \partial/\partial x_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \partial/\partial x_1 & \partial/\partial x_2 \end{bmatrix}^T \tag{5}$$

In Eq. (4), $\mathbf{C}$ is the elasticity matrix, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$ the matrices of dielectric constants and magnetic permeability, respectively, $\mathbf{e}$ and $\mathbf{d}$ the matrices of piezoelectric and piezomagnetic constants and $\mathbf{g}$ the matrix describing the direct magneto-electric coupling. Eqs. (1)–(3) formally resemble the governing equations of anisotropic elasticity. They can be rearranged to obtain the governing equations of the magneto-electro-elasticity, analogous to Navier's equations for elasticity

$$\mathcal{D}^T\mathbf{R}\mathcal{D}\mathbf{U}+\mathbf{F}=\mathbf{0} \quad \text{in } \Omega \tag{6}$$

For the dynamic problem, the generalized body forces term is given by the sum of the inertial forces and the generalized applied loads $\mathbf{q}$. It can be written

$$\mathbf{F}=\begin{bmatrix} f_1 \\ f_2 \\ -\omega \\ 0 \end{bmatrix}=\begin{bmatrix} q_1 \\ q_2 \\ -\omega \\ 0 \end{bmatrix}-\rho\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ 0 \\ 0 \end{bmatrix}=\mathbf{q}-\boldsymbol{\rho}\ddot{\mathbf{U}} \tag{7}$$

where the overdot indicates the time derivative and the matrix $\boldsymbol{\rho}$ denotes the product of the material density by the 4×4 identity matrix in which the last two diagonal terms are replaced by zeros. It is worth noting that, with this definition, the generalized inertia term takes into account the quasi-static modelling of electric and magnetic fields.

In the generalized variable notation essential and natural boundary conditions, associated with Eq. (6), are expressed as

$$\mathbf{U}=\overline{\mathbf{U}} \quad \text{on } \partial\Omega_1 \tag{8}$$

$$\mathbf{T}=\mathcal{D}_n^T\mathbf{R}\mathcal{D}\mathbf{U}=\overline{\mathbf{T}} \quad \text{on } \partial\Omega_2 \tag{9}$$