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Regularized meshless method analysis of the problem of obliquely incident water wave

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ABSTRACT

In this paper, an application of regularized meshless method (RMM) for solving the problem of obliquely incident water wave passing a submerged breakwater is presented. By using desingularization technique to regularize the singularity and hypersingularity of the kernel functions, source points can be located on the physical boundary of an arbitrary domain. To verify the practicability and validity of the RMM, simulations for observing the propagation of oblique incident wave through a barrier are presented where the modified Helmholtz equation is satisfied. Finally, three examples are given to show the effects of breakwater with rigid and absorbing boundary conditions to energy dissipation caused by existence of a barrier. After comparing such analytical solution with the corresponding boundary element method (BEM) solutions, they are shown to be in good agreement.

1. Introduction

In the past decade, meshless method, so-called mesh free method, has been a well-known numerical method, and it has been a popular method for scientific computing due to a strong demand of taking a fewer time on mesh generation in domains of high dimensions. The model setup in the process of using meshless methods takes less time than that in using boundary element method. It is expected that meshless methods will be a significant and promising alternative that dominates future numerical computations. Several important types of meshless methods are reported in literatures [3–5,10–15,17,18,21,22,24,27,29,37].

Among the aforesaid meshless methods, method of fundamental solutions (MFS) has been extensively applied to solving engineering problems [4,17,29], in which this method is related to an indirect method of single-layer potential. MFS is one kind of meshless method in which only boundary nodes are needed. Comprehensive reviews of the MFS were published by Fairweather and Karageorghis [17] and Chen and Golberg [4]. In order to avoid the problem of singularity, the sources points of a set of single layer potential (corresponding to the fundamental solution) are located on nonphysical boundary (namely fictitious boundary). A singularity-free method with regular formulation is then obtained. It is effective and relatively easy to use. However, the MFS has not become a popular numerical method because of some controversy from making an artificial selection of off-set distance between artificial boundary and physical boundary. In general, it is difficult to choose an optimal fictitious boundary in a complicated geometry. This brings some limitation to the implementation of MFS, since the appropriate location of source points requires accurate estimation. The diagonal coefficients of the influence matrices are divergent in common cases when the corresponding fictitious boundary approaches the real boundary. Despite of the disappearance of singularities, the influence matrices become ill-posed when the fictitious boundary is far away from the real boundary. The results become very unstable, since the condition number gets very large.

Aim of this paper is to propose the developed meshless method [35], the regularized meshless method (RMM), for solving modified Helmholtz equation, where the source points are located on the physical boundary. We present an alternative to the traditional numerical approach by retaining the salient meshless features of MFS and taking normal derivative of the fundamental solution of modified Helmholtz equation as radial basis functions (RBF) [4,5,12,34,35]. The solution is expressed in terms of a double-layer potential instead of a single-layer potential on the physical boundary without having an integration process. Through the application of desingularization technique upon the diagonal terms when the source point and boundary points are coincident, the proposed meshless method can avoid the occurrence of ambiguity of off-set distance in the conventional MFS.

By using the regularization technique of subtracting and adding-back, the singularity and hypersingularity of the kernel functions can be regularized. The main idea is to add an augmented series

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containing one singular term and then subtract the same from the representing series of solution. The two singular terms are equal to each other. At the same time, the zero sum of the augmented series can be derived in the proposed formulation [7,12,34,35] and may benefit the regularization process during the formulation. In general, the diagonal terms of the influence matrices can then be derived for arbitrary domain. Furthermore, the innovative concept of this study is that this regularization technique has been of little use within meshless methods except in recent publications on Laplace and Helmholtz equation [7–9,12,34,35]. In this study, we extend the Helmholtz equation considered in the above publications to modified Helmholtz equation. For the final purpose of demonstrating practicability and validity of the method, we present several test problems for 2-D scattering water waves through submerged breakwater, governed by a modified Helmholtz equation.

The primary function of submerged breakwaters is to reduce wave energy transmitted through it and to have the advantages of allowing water circulation, fish passage, and provision of economical protection. A suitable thin barrier may act as a good model for breakwater. Prediction of wave interactions has been studied previously for many kinds of configuration of water barrier in linear wave diffraction theory [2,6,16,19,20,23,25,26,28,29–33,36]. Many analytical and numerical solutions have been developed to solve the water wave problem, such as the eigenfunction expansion method [18,25,26] and the boundary element method [6,23,28]. The reflection and transmission of obliquely incident water wave past a submerged barrier with a finite width were studied using boundary element method (BEM) in linear wave theory [5,22]. In this paper, we implement the RMM to solve the problems of obliquely incident water wave to demonstrate the practicability of our proposed method. The boundary condition of breakwater can be the absorbing boundary condition with different absorbing parameters on both front and back sides of the breakwater. The results will be compared with those obtained by simulation and by using eigenfunction expansion method.

2. Formulation

2.1. Governing equation

The real physical problems for the modified Helmholtz equation constrain the 2-D water wave scattering problem through the submerged breakwater problem as follows:

Consider a vertical thin barrier paralleled to the z-axis as shown in Fig.1. A wave train with a frequency σ propagates towards the barrier with an angle θ in a constant water depth h. Assuming there is an inviscid, incompressible fluid and irrotational flow, the wave field may be represented by the velocity potential $\Phi(x,y,z,t)$ which satisfies the Laplace equation as

$$\nabla^2 \Phi(x,y,z,t) = 0 \tag{1}$$

According to the uniformity of the water depth in the z-axis and the periodicity in time, the potential $A^{(i)}(s^i,x^i)$ of fluid motion can be expressed as

$$\Phi(x,y,z,t) = \phi(x,y)\,e^{(\lambda z - \sigma t)i} \tag{2}$$

where $\lambda = k\sin(\theta)$ and k is the wave number which satisfies the dispersion relation:

$$\sigma^2 = gk\tanh(kh) \tag{3}$$

where g is the acceleration of gravity. The unknown function, $\phi(x,y)$, describes the fluctuation of the potential on the x–y plane. Substitution of Eq. (2) into Eq. (1) yields the modified Helmholtz equation as follows:

$$\nabla^2 \phi(x,y) - \lambda^2 \phi(x,y) = 0, \quad (x,y) \in D \tag{4}$$

where D is the domain of interest.

2.2. Boundary conditions

The boundary conditions (BCs) of the interested domain are summarized as

1. *The linearized free water surface boundary condition*:

$$\frac{\partial \phi(x,y)}{\partial y} - \frac{\sigma^2 \phi(x,y)}{g} = 0 \tag{5}$$

2. *Seabed and breakwater boundary conditions*:
   (a) Rigid boundary condition:

$$\frac{\partial \phi(x,y)}{\partial n} = 0 \tag{6}$$

   where n is boundary normal vector.
   (b) Absorbing boundary condition:

$$\frac{\partial \phi_1(x,y)}{\partial n} = ikG_1\phi_1(x,y) \tag{7}$$

$$\frac{\partial \phi_2(x,y)}{\partial n} = -ikG_2\phi_2(x,y) \tag{8}$$

   where $\phi_1(x,y)$ and $\phi_2(x,y)$ are the potential of both front and back sides of the breakwater and G_1 and G_2 are the corresponding absorbing parameters, respectively.

3. *Radiation condition at infinity*:

$$\lim_{x\to\infty} x^{1/2}\left[\frac{\partial \phi(x,y)}{\partial x} - ik\phi(x,y)\right] = 0 \tag{9}$$

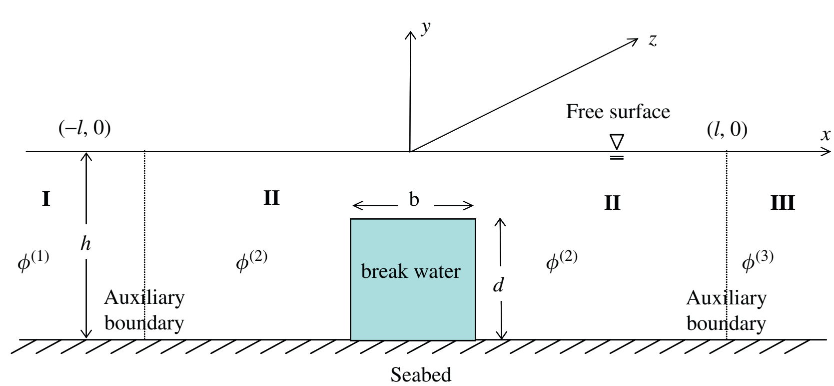

Fig. 1. Definition sketch of the water scattering problem of oblique incident wave past a breakwater.