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Law of large numbers for random walks on attractive spin-flip dynamics

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Abstract

We prove a law of large numbers for certain random walks on certain attractive dynamic random environments when initialised from all sites equal to the same state. This result applies to random walks on $\mathbb{Z}^d$ with $d \geq 1$. We further provide sufficient mixing conditions under which the assumption on the initial state can be relaxed, and obtain estimates on the large deviation behaviour of the random walk.

As prime example we study the random walk on the contact process, for which we obtain a law of large numbers in arbitrary dimension. For this model, further properties about the speed are derived.
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1. Introduction and main results

1.1. Background and outline

Random walks in random environment (RWRE) gained much interest throughout the last decades. Such models serve as natural extensions of the classical random walk model and have broad applications in physics, chemistry and biology.

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RWRE models show significantly different behaviours than the simple random walk model. This was already observed in one of the first models studied, Solomon [25], where it was shown that the random walk can behave sub-ballistically. Non-Gaussian scaling limits were established for the same model in Kesten, Kozlov, and Spitzer [15] and Sinaĭ [24]. These characteristics are due to trapping phenomena.

RWRE models on $\mathbb{Z}$ are by now well understood in great generality whenever the environment is static, i.e. it does not change with time. On the other hand, for RWRE models on $\mathbb{Z}^d$, $d \geq 2$, the analysis of trapping phenomena becomes much more delicate and less is known. See for instance Zeitouni [27] or Sznitman [26] for an overview of results, and Drewitz and Ramírez [7] for a monograph with focus on recent developments.

In the last decade, much focus has been devoted to models where the random environment evolves with time, i.e. random walks in *dynamic* random environments (RWDRE). It is believed that the extent to which trapping phenomena occur for RWDRE models depends on the correlation structure of the dynamics.

At a rigorous level, it is known to great generality that RWDRE models scale diffusively when the environment is only weakly correlated in space–time; see for instance Redig and Völlering [22]. These results are not restricted to random walks on $\mathbb{Z}$, but are valid in any dimension. Here weakly correlated essentially means that the environment becomes approximately independent of its starting configuration within a space–time cone, also known as cone mixing environment.

Little is known at a general level when the environment has a non-uniform correlation structure, though trapping phenomena are conjectured to occur for some specific models (Avena and Thomann [4]). Avena, den Hollander and Redig [1] have shown rigorously that a random walk on the one-dimensional exclusion process exhibits trapping phenomena at the level of large deviations under drift assumptions. On the other hand, several other models with a non-uniform correlation structure have been shown to possess diffusive scaling limits, for example Avena, dos Santos and Völlering [3], Hilário, den Hollander, Sidoravicius, dos Santos, and Teixeira [11], den Hollander and dos Santos [12], Huveneers and Simenhaus [14] and Mountford and Vares [19].

In this paper we present a strong law of large numbers (SLLN) for random walks on certain attractive (or monotone) interacting particle system (IPS). For this, restrictions on both the random walk and the IPS are required. In particular, we assume that the sites of the IPS take values 0 or 1 and that the IPS has a graphical representation coupling which is monotone with respect to the initial configuration. One class of IPS satisfying the latter assumption is additive and attractive spin-flip systems.

The SLLN is obtained when the IPS is initialised at time 0 from a configuration where all sites have the same value, assuming in addition that the jump transitions of the random walk only depend on the state of the IPS at the position of the random walk. Under certain mixing conditions, we are able to relax the restriction on the starting configuration.

An important feature of the SLLN is that it does not rely directly on the correlation structure of the environment, but rather assumes monotonicity. In particular, the SLLN applies to a large class of models with non-uniform correlation structure not previously considered in the literature. Furthermore, the SLLN applies to random walks on $\mathbb{Z}^d$ for any $d \geq 1$ and is not restricted to nearest neighbour jumps.

We also provide large deviation estimates for the random walk. In particular, we show that no trapping phenomena occur for our model at the level of large deviations throughout the cone mixing regime.

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