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Percolation of even sites for enhanced random sequential adsorption

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Abstract

Consider random sequential adsorption on a chequerboard lattice with arrivals at rate 1 on light squares and at rate λ on dark squares. Ultimately, each square is either occupied, or blocked by an occupied neighbour. Colour the occupied dark squares and blocked light sites *black*, and the remaining squares *white*. Independently at each meeting-point of four squares, allow diagonal connections between black squares with probability p; otherwise allow diagonal connections between white squares. We show that there is a critical surface of pairs (λ, p), containing the pair $(1, 0.5)$, such that for (λ, p) lying above (respectively, below) the critical surface the black (resp. white) phase percolates, and on the critical surface neither phase percolates.

MSC: 60K35; 82B43

Keywords: Dependent percolation; Random sequential adsorption; Critical surface

1. Introduction

Random sequential adsorption (abbreviated RSA throughout this paper) is a term for a family of probability models for irreversible particle deposition. Particles arrive at random locations and times onto a surface, and if accepted a particle blocks nearby locations on the surface from

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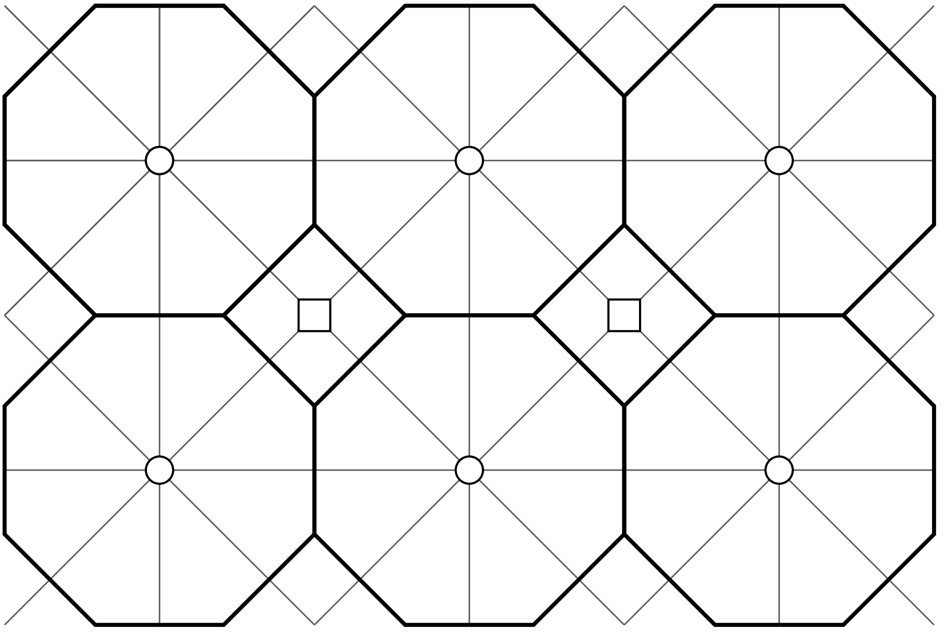

Fig. 1. A section of the lattice used for enhanced RSA and the associated tiling of $\mathbb{R}^2$. The small circles represent octagon sites and the small squares are diamond sites.

accepting future arrivals. Such models are of physical interest, as a model for coating of a surface; see for example [5,13]. We consider a discrete version of RSA on the initially empty integer lattice $\mathbb{Z}^2$, with the arrival time at a lattice site x given by an exponential random variable T_x with parameter λ_x, with $(T_x)_{x\in\mathbb{Z}^2}$ independent. All sites are either *empty*, *occupied* or *blocked*; an arrival at an empty site x causes it to become permanently occupied and all adjacent sites (that is, sites y such that $|x-y|=1$ where $|\cdot|$ denotes the Euclidean norm) to become permanently blocked. If $\sup_x \lambda_x < \infty$ this model is well defined; see [10]. On this lattice we define the even (respectively, odd) sites to be those at an even (respectively, odd) graph distance from the origin.

Ultimately, each site will be either occupied or blocked. The distribution of the occupied and blocked sites in this ultimate state is called the jamming distribution; under the jamming distribution the sites of $\mathbb{Z}^2$ are divided into an even phase and an odd phase, where the even phase consists of occupied even sites and blocked odd sites. Site percolation of the even phase was considered in [11], in the case where for some $\lambda > 0$ we have $\lambda_x = 1$ for odd x and $\lambda_x = \lambda$ for even x. The even phase is *monotone* in λ; that is, for $0 \le \lambda < \lambda'$ there exists coupled realizations of the process just described with parameter λ and with parameter λ', such that the even phase for parameter λ is contained in the even phase for parameter λ'.

Penrose and Rosoman [11] proved that the critical parameter λ for RSA on the integer lattice $\mathbb{Z}^2$ is strictly greater than 1. The proof of this uses an *enhanced RSA* (denoted eRSA below) model on a new lattice called Λ throughout this paper. We associate with each site $x \in \mathbb{Z}^2$ a site $x' := x + (1/2, 1/2)$. The lattice Λ has vertex set $\cup_{x\in\mathbb{Z}^2}\{x, x'\}$, with an edge between sites $x \in \mathbb{Z}^2$ and $y \in \mathbb{Z}^2$ if $|x-y| = 1$, and an edge between x' and y if $|x'-y| = \frac{\sqrt{2}}{2}$ (here $|\cdot|$ is the Euclidean distance). We refer to the added sites x' as diamond sites, and the original sites x as octagon sites; each octagon site has degree 8 and each diamond site has degree 4 (see Fig. 1).