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Least squares estimators for stochastic differential equations driven by small Lévy noises

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Highlights

- We consider parameter estimation for stochastic processes driven by Lévy noises.
- We propose least squares estimator for the drift parameters.
- Consistency and rate of convergence of the estimator are established.
- A simulation study illustrates the asymptotic behavior of the estimator.

Abstract

We study parameter estimation for discretely observed stochastic differential equations driven by small Lévy noises. We do not impose Lipschitz condition on the dispersion coefficient function σ and any moment condition on the driving Lévy process, which greatly enhances the applicability of our results to many practical models. Under certain regularity conditions on the drift and dispersion functions, we obtain consistency and rate of convergence of the least squares estimator (LSE) of parameter when $\varepsilon \to 0$ and $n \to \infty$ simultaneously. We present some simulation study on a two-factor financial model driven by stable noises.
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1. Introduction

Let $\mathbb{R}_0^r = \mathbb{R}^r \backslash 0$ and let $\mu(du)$ be a σ-finite measure on $\mathbb{R}_0^r$ satisfying $\int_{\mathbb{R}_0^r}(|u|^2 \wedge 1)\mu(du) < \infty$ with $|u| = (\sum_{i=1}^r u_i^2)^{1/2}$. Let $\{B_t = (B_t^1, \ldots, B_t^r) : t \geq 0\}$ be an r-dimensional standard Brownian motion and let $N(dt, du)$ be a Poisson random measure on $(0, \infty) \times \mathbb{R}_0^r$ with intensity measure $dt\mu(du)$. Suppose that $\{B_t\}$ and $\{N(dt, du)\}$ are independent of each other. Then an r-dimensional Lévy process $\{L_t\}$ can be given as

$$L_t = B_t + \int_0^t \int_{|u|>1} uN(ds, du) + \int_0^t \int_{|u|\leq 1} u\tilde{N}(ds, du), \tag{1.1}$$

where $\tilde{N}(ds, du) = N(ds, du) - ds\mu(du)$. Let us consider a family of d-dimensional jump–diffusion processes defined as the solution of

$$\begin{aligned} dX_t^\varepsilon &= b(X_t^\varepsilon, \theta)dt + \varepsilon\sigma(X_{t-}^\varepsilon)dL_t, \quad t \in [0, 1], \\ X_0^\varepsilon &= x, \end{aligned} \tag{1.2}$$

where $\theta \in \bar{\Theta}$, the closure of an open convex bounded subset Θ of $\mathbb{R}^p$. The function $b(x, \theta) = (b^k(x, \theta))$ is $\mathbb{R}^d$-valued and defined on $\mathbb{R}^d \times \bar{\Theta}$; the function $\sigma(x) = (\sigma_l^k(x))$ is defined on $\mathbb{R}^d$ and takes values on the space of matrices $\mathbb{R}^d \otimes \mathbb{R}^r$; the initial value $x \in \mathbb{R}^d$, and $\varepsilon > 0$ are known constants. A stochastic process of form (1.2) has long been used in the financial world and has been the fundamental tool in financial modeling. We refer to Sundaresan [38] and Fan [7] for overviews, Barndorff-Nielsen, Mikosch and Resnick [2] for recent developments on Lévy-driven processes, and Sørensen [34], Shimizu and Yoshida [33], Shimizu [30], Ogihara and Yoshida [26] and Masuda [25] for statistical inference. Examples of (1.2) include (i) the multivariate diffusion process defined by

$$dX_t^\varepsilon = b(X_t^\varepsilon, \theta)dt + \varepsilon\sigma(X_t^\varepsilon)dB_t,$$

see Stroock and Varadhan [37]; (ii) the Vasicek model with jumps or the Lévy driven Ornstein–Uhlenbeck process defined by

$$dX_t^\varepsilon = \kappa(\beta - X_t^\varepsilon)dt + \varepsilon dL_t,$$

where κ and β are positive constants, and L_t can be chosen to be a standard symmetric α-stable Lévy process (see Hu and Long [12] and Fasen [8]) or a (positive) Lévy subordinator (see Barndorff-Nielsen and Shephard [3]); (iii) the Cox–Ingersoll–Ross (CIR) model driven by α-stable Lévy processes defined by

$$dX_t^\varepsilon = \kappa(\beta - X_t^\varepsilon)dt + \varepsilon \sqrt[\alpha]{X_{t-}^\varepsilon}\, dL_t, \tag{1.3}$$

where $\{L_t\}$ is a spectrally positive α-stable process with $1 < \alpha < 2$; see Fu and Li [9] and Li and Ma [18].

Assume that the only unknown quantity in (1.2) is the parameter θ. We denote the true value of the parameter by θ_0 and assume that $\theta_0 \in \Theta$. Suppose that this process is observed at regularly spaced time points $\{t_k = k/n,\ k = 1, 2, \ldots, n\}$. The purpose of this paper is to study the least squares estimator for the true value θ_0 based on the sampling data $(X_{t_k})_{k=1}^n$ with small dispersion ε and large sample size n. There are several practical advantages in small noise asymptotics:(i) we can get the drift parameter estimation by samples from a fixed finite time interval under relatively

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