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Supercritical loop percolation on $\mathbb{Z}^d$ for $d \geq 3$

Yinshan Chang*

Department of Mathematics, University of Sichuan, Chengdu, China
Max-Planck-Institute for Mathematics in the Sciences, Leipzig, Germany

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Abstract

We consider supercritical percolation on $\mathbb{Z}^d$ ($d \geq 3$) induced by random walk loop soup. Two vertices are in the same cluster if they are connected through a sequence of intersecting loops. We obtain quenched parabolic Harnack inequalities, Gaussian heat kernel bounds, the invariance principle and the local central limit theorem for the simple random walks on the unique infinite cluster. We also show that the diameter of finite clusters have exponential tails like in Bernoulli bond percolation. Our results hold for all $d \geq 3$ and all supercritical intensities despite polynomial decay of correlations.
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1. Introduction

We consider the simple random walk loop soup on $\mathbb{Z}^d$ for $d \geq 3$. We start with a description of the model. As in [21], an element $\dot{\ell} = (x_1, \ldots, x_n)$ of V^n, $n \geq 2$, satisfying $x_1 \neq x_2, \ldots, x_n \neq x_1$ is called a non-trivial discrete based loop. We define its *length* $|\dot{\ell}|$ to be n. Two based loops of length n are equivalent if they coincide after a circular permutation of their coefficients, i.e. $(x_1, \ldots, x_n)$ is equivalent to $(x_i, \ldots, x_n, x_1, \ldots, x_{i-1})$ for all i. Equivalence classes of non-trivial discrete based loops for this equivalence relation are called (non-trivial) discrete loops. For a loop ℓ (equivalence class of $\dot{\ell}$), we define its *length* $|\ell|$ to be $|\dot{\ell}|$.

* Correspondence to: Department of Mathematics, University of Sichuan, Chengdu, China.
E-mail address: ychang@scu.edu.cn.

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We associate to each based loop $\dot{\ell} = (x_1, \dots, x_n)$ the weight

$$\dot{\mu}(\dot{\ell}) = \begin{cases} \frac{1}{n}(2d)^{-n} & \text{if } x_1, x_2, \dots, x_n, x_1 \text{ is a nearest neighbor path,} \\ 0 & \text{otherwise.} \end{cases}$$

The push-forward of $\dot{\mu}$ on the space of discrete loops is denoted by μ. The random walk loop soup of intensity $\alpha > 0$, denoted by $\mathcal{L}_\alpha$, is the Poisson point process of loops of intensity $\alpha\mu$.

The random walk loop soup naturally induces a percolation process on edges of $\mathbb{Z}^d$, which we call *loop percolation*. In this model, an edge $\{x, y\}$ is called open if it is crossed by at least one loop $\ell \in \mathcal{L}_\alpha$ in any direction. Open edges form clusters of vertices. We denote by $\mathcal{C}_\alpha(x)$ the open cluster containing x. Let $\alpha_c = \inf\{\alpha > 0 : \mathbb{P}[\#\mathcal{C}_\alpha(0) = \infty] > 0\}$ be the *critical threshold* of the loop percolation.

The random walk loop soup on $\mathbb{Z}^2$ was introduced by G.F. Lawler and J.A. Trujillo Ferreras [18] as a discrete analogue and approximation of Brownian loop soup of Lawler and W. Werner [19]. The random walk loop soups on arbitrary graphs were extensively studied by Y. Le Jan [20]. The percolation of loops was first considered by Lawler and Werner in [19] and by S. Sheffield and Werner [31] in the setting of the two dimensional Brownian loop soup, who among other results, identified the value of the critical intensity. Recently, these results were used by T. Lupu [22] to identify the value of the critical intensity of the loop percolation on the discrete half plane. The first study of loop percolation on $\mathbb{Z}^d$ was done by Le Jan and S. Lemaire [21], who proved that $\alpha_c < \infty$ for $\mathbb{Z}^d$ by dominating the loop percolation from below by Bernoulli bond percolation. The positivity of α_c was proved in [9,23] for $d \geq 3$. In fact, [23] proves that the critical threshold α_c is above $1/2$ using a coupling with the Gaussian free field. A first comprehensive study of the loop percolation on $\mathbb{Z}^d$ was realized in [9]. In particular, it was proved that the loop percolation is invariant and ergodic with respect to lattice shifts ([9, Proposition 3.2]) and has polynomial decay of correlations ([9, Proposition 3.1][1]). Also, [9] gives a detailed analysis of the subcritical phase. In this paper, we obtain a rather complete understanding of the supercritical phase. We prove that many properties of finite clusters and of the unique infinite cluster are similar to those of supercritical Bernoulli percolation. In contrast, it is shown in [9] that various characteristics of clusters in subcritical regime have polynomial tails showing a very different behavior from subcritical Bernoulli percolation.

For finite clusters in the supercritical regime, we obtain upper bounds of the decay of diameters of finite clusters.

Theorem 1.1. *For $d \geq 3$ and $\alpha > \alpha_c$, there exist constants $c = c(d, \alpha) > 0$, $C = C(d, \alpha) < \infty$ such that $\forall n \geq 1$,*

$$\mathbb{P}[\#\mathcal{C}_\alpha(0) < \infty, \mathcal{C}_\alpha(0) \cap \partial B(n) \neq \emptyset] \leq C(d, \alpha)e^{-c(d,\alpha)n},$$

where $B(n) = \{x \in \mathbb{Z}^d : \|x\|_\infty \leq n\}$ and $\partial B(n) = \{x \in \mathbb{Z}^d : \|x\|_\infty = n\}$.

For simple random walks on the unique infinite cluster, we have

Theorem 1.2. *For $d \geq 3$ and $\alpha > \alpha_c$, we consider the discrete time simple random walk $(X_n)_{n\geq 0}$ on the infinite cluster $\mathcal{S}_\infty$. For $x \in \mathcal{S}_\infty$, let*

$$\nu_x = \sum_{y \in \mathcal{S}_\infty} 1_{\{y,x\} \text{ is an edge in } \mathcal{S}_\infty}. \tag{1}$$

[1] Although it was stated for sites there, similar asymptotics hold for edges.

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