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Ergodic decompositions of stationary max-stable processes in terms of their spectral functions

Clément Dombry[a], Zakhar Kabluchko[b,*]

[a] *Univ. Bourgogne Franche-Comté, Laboratoire de Mathématiques de Besançon, UMR CNRS 6623, 16 route de Gray, 25030 Besançon cedex, France*

[b] *Universität Münster, Institut für Mathematische Statistik, Orléans-Ring 10, 48149 Münster, Germany*

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Abstract

We revisit conservative/dissipative and positive/null decompositions of stationary max-stable processes. Originally, both decompositions were defined in an abstract way based on the underlying non-singular flow representation. We provide simple criteria which allow to tell whether a given spectral function belongs to the conservative/dissipative or positive/null part of the de Haan spectral representation. Specifically, we prove that a spectral function is null-recurrent iff it converges to 0 in the Cesàro sense. For processes with locally bounded sample paths we show that a spectral function is dissipative iff it converges to 0. Surprisingly, for such processes a spectral function is integrable a.s. iff it converges to 0 a.s. Based on these results, we provide new criteria for ergodicity, mixing, and existence of a mixed moving maximum representation of a stationary max-stable process in terms of its spectral functions. In particular, we study a decomposition of max-stable processes which characterizes the mixing property.

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* Corresponding author.
E-mail addresses: clement.dombry@univ-fcomte.fr (C. Dombry), zakhar.kabluchko@uni-muenster.de (Z. Kabluchko).

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1. Statement of main results

1.1. Introduction

A stochastic process $(\eta(x))_{x\in\mathcal{X}}$ on $\mathcal{X}=\mathbb{Z}^d$ or $\mathcal{X}=\mathbb{R}^d$ is called *max-stable* if

$$\frac{1}{n}\bigvee_{i=1}^{n}\eta_i \overset{f.d.d.}{=} \eta \quad \text{for all } n\geq 1,$$

where $\eta_1,\dots,\eta_n$ are i.i.d. copies of η, $\bigvee$ is the pointwise maximum, and $\overset{f.d.d.}{=}$ denotes the equality of finite-dimensional distributions. Max-stable processes arise naturally when considering limits for normalized pointwise maxima of independent and identically distributed (i.i.d.) stochastic processes and hence play a major role in spatial extreme value theory; see, e.g., de Haan and Ferreira [4]. We restrict our attention to processes with non-degenerate (non-constant) margins. The above definition implies that the marginal distributions of η are 1-Fréchet, that is

$$\mathbb{P}[\eta(x)\leq z]=\mathrm{e}^{-c(x)/z} \quad \text{for all } z>0,$$

where $c(x)>0$ is a scale parameter.

A fundamental representation theorem by de Haan [3] states that any stochastically continuous max-stable process η can be represented (in distribution) as

$$\eta(x)=\bigvee_{i\geq 1}U_iY_i(x), \quad x\in\mathcal{X}, \tag{1}$$

where

- $(U_i)_{i\geq 1}$ is a decreasing enumeration of the points of a Poisson point process on $(0,+\infty)$ with intensity measure $u^{-2}\mathrm{d}u$,
- $(Y_i)_{i\geq 1}$, which are called the *spectral functions*, are i.i.d. copies of a non-negative process $(Y(x))_{x\in\mathcal{X}}$ such that $\mathbb{E}[Y(x)]<+\infty$ for all $x\in\mathcal{X}$,
- the sequences $(U_i)_{i\geq 1}$ and $(Y_i)_{i\geq 1}$ are independent.

In this paper, we focus on *stationary* max-stable processes that play an important role for modelling purposes; see, e.g., Schlather [21]. The structure of stationary max-stable processes was first investigated by de Haan and Pickands [5] who related them to non-singular flows (which are referred to as "pistons" in [5]). Using the analogy between max-stable and sum-stable processes and the works of Rosiński [13,14], Rosiński and Samorodnitsky [15] and Samorodnitsky [19,20] on sum-stable processes, the representation theory of stationary max-stable processes via non-singular flows was developed by Kabluchko [7], Wang and Stoev [26,25], Wang et al. [24]. In these papers, the conservative/dissipative (or Hopf) and positive/null (or Neveu) decompositions from non-singular ergodic theory were used to introduce the corresponding decompositions $\eta=\eta_C\vee\eta_D$ and $\eta=\eta_P\vee\eta_N$ of the stationary max-stable process. These definitions were rather abstract (see Sections 3 and 4 where we shall recall them) and did not allow to distinguish between conservative/dissipative or positive/null cases by looking just at the spectral functions Y_i from the de Haan representation (1). The purpose of this paper is to provide a *constructive* definition of these decompositions. Our main results in this direction can be summarized as follows. In Section 3 we shall prove that in the case when the sample paths of η are a.s. locally bounded, a spectral function Y_i belongs to the dissipative (=mixed moving maximum) part of the process