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BEM simulations of potential flow with viscous effects as applied to a rising bubble

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ABSTRACT

A model for the unsteady rise and deformation of non-oscillating bubbles under buoyancy force at high Reynolds numbers has been implemented using a boundary element method. Results such as the evolution of the bubble shape, variations of the transient velocity with rise height and the terminal velocity for different size bubbles have been compared to recent experimental data in clean water and to numerical solutions of the unsteady Navier–Stokes equation. The aim is to capture the essential physical ingredients that couple bubble deformation and the transient approach towards terminal velocity. This model requires very modest computational resources and yet has the flexibility to be extended to more general applications.

1. Introduction

Results of experiments pertaining to bubble rise in a variety of liquids and liquid mixtures have been reviewed recently by Loth [1]. Magnaudet and Eames [2] summarised various theoretical and empirical approaches to the problem of bubble motion at high Reynolds numbers. The focus of this paper is on modelling the behaviour of rising bubbles in clean water for equivalent bubble diameters up to ~ 4 mm or Reynolds number up to ~ 1000. The aim is to predict the transient rise velocity and corresponding deformation of initially spherical bubbles under the influence of buoyancy forces. The well-known summary of terminal velocity, U, versus equivalent bubble diameter, d, in clean and unpurified water of Clift et al. [3] (Fig. 1) provides a good visual overview. Also shown in this figure are theoretical results for a spherical bubble corresponding to the Hadamard–Rybczynski (HR) formula [4]: $U = 4\rho g d^2/3\mu$ valid for Stokes flow; the Levich formula [5–7]: $U = 4\rho g d^2/9\mu$ valid in the limit of infinite Reynolds number, $Re = \rho U d/\mu \to \infty$; and the empirical correlation formula given by Magnaudet and Eames [2] which has been constructed from the results of Mei et al. [8] and Moore [9] (together denoted as MM) to fit experimental data for spherical bubbles in the range $0 \le Re \le 500$. Here ρ is the water density, μ the dynamic shear viscosity and g the gravitational acceleration. All these theoretical results assume the zero tangential stress boundary condition at the surface of a spherical bubble which is appropriate for experiments conducted in highly purified water. It is well known that rising bubbles in ultra clean water follow a rectilinear path until a critical equivalent diameter of about 2 mm when the terminal velocity attains a local maximum, and beyond which the bubble path can zig-zag or spiral. However, the terminal velocity in the rectilinear regime is very sensitive to even trace amounts of contamination so it is important to cross-validate experimental data from a number of independent sources and ascertain that the measurements are free from artifacts.

Recently, Malysa et al. [10] measured the transient rise of deformed bubbles of equivalent diameter between 1.35 and 1.43 mm (around the location of the velocity maximum in Fig. 1) in ultra clean water. The observed terminal velocity of 35 cm/s (which corresponds to a Reynolds number of about 500) is in excellent agreement with the experiments of Duineveld [11] and Wu and Gharib [12] who measured the terminal velocities of deformed bubbles in clean water in the range of equivalent diameters between 1 and 2 mm. These results are also consistent with the earlier measurements of Okazaki [13]. This range of bubble size is of particular significance in mineral flotation applications [14] and many other industrial processes.

With small amounts of added surfactants, Malysa et al. [10] reported that the terminal velocity for same sized bubbles fell to 15 cm/s, which again is in excellent agreement with the observations of Zhang and Finch [14] and of Wu and Gharib [12] for cases where the bubbles become contaminated by the bubble generation method. These small concentrations of added surfactants render the bubble surface immobile while the interfacial tension remains unaffected.

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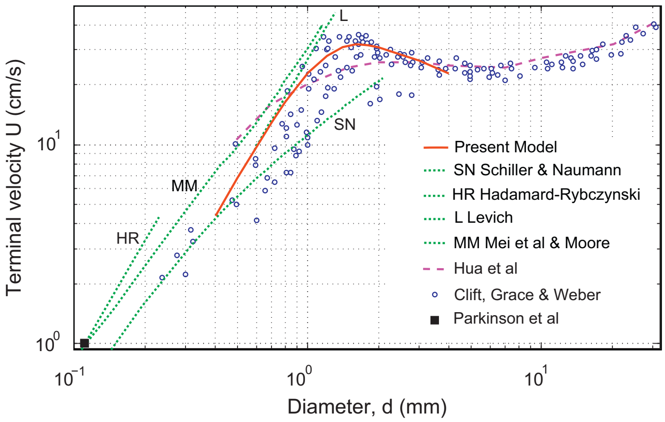

Fig. 1. Numerical results of the terminal velocity of deformable bubbles predicted by the present model and by the full 3D solution of the unsteady Navier–Stokes equation of Hua et al. are compared to experimental data summarised by Clift et al. [3]. Familiar theoretical results for spherical bubbles are also plotted. See text for details.

The results of three independent experimental studies by Duineveld, Malysa et al. and Wu and Gharib on the terminal oblate ellipsoidal shapes of bubbles in clean water as characterized by variations of the aspect ratio with equivalent bubble size are also in excellent agreement (see Fig. 2b). In the presence of surfactants or for contaminated bubbles, all three studies reported lower terminal velocities of the same magnitude ($Re \sim 200$) and the terminal bubble shapes remain nearly spherical. Therefore, one can be quite confident that these results represent the correct behaviour of rising bubbles in water under clean, contaminant free conditions. Taken together with the recent measurements of the rise of microbubbles in ultra clean water by Parkinson et al. [15], we have a complete and reliable experimental data set for the rectilinear rise of spherical and deformed bubbles in water for Reynolds numbers up to ~ 500.

Numerical modelling of unsteady rising bubbles using grid-based numerical methods that take into account deformations in a self-consistent way has been attempted using both an axi-symmetric boundary-fitted coordinate formulation [16,17] and a full three-dimensional solution [18,19] of the Navier–Stokes equation. Such approaches are quite complex to implement and are very demanding in terms of computational resources [20]. This places practical limitations on extending them to more complex and interesting multiphase problems [21] involving, for example the motion and deformation of multiple bubbles in response to external fields or to model dynamic interactions between bubbles and between bubbles and surfaces or interfaces. A relatively simple, yet accurate model that can accommodate these complexities at relatively high Reynolds numbers is therefore desirable [10].

A promising approach to treat bubble dynamics at high Reynolds number is via a boundary integral formulation that only uses the properties of the bubble surface to track its evolution. In addition to computational efficiencies conferred by the reduction of one spatial dimension, the focus on the boundary means that interactions between bubbles and surfaces that may involve short-ranged surface forces can be included without complex implementation issues associated with obtaining a sufficiently accurate resolution of a deforming air/water interface in grid-based computational schemes.

The boundary integral method has been used in the past to simulate models involving deformable rising bubbles. Miksis et al. [22] considered potential flow and obtained shapes and terminal velocities but did not consider transient behavior. Boulton-Stone et al. [23] and Blake et al. [24] considered the transient motion of one bubble or a pair of bubbles rising in the absence of viscosity effects and so did not address the question of terminal velocities.

The theory considered in this paper is motivated by the boundary integral formulation by Lundgren and Mansour [25], appropriate at high Reynolds numbers, to study weak viscous effects on the oscillation of a liquid drop in a gravity-free environment. However, for the rising bubble problem, we include a gravitational body force at the outset. The aim is to produce a theory that can describe the evolution of the position, velocity and deformations of the bubble surface in a self-consistent way. At high Reynolds number, the viscous potential flow approach [26] is able to predict the exact limiting forms of the terminal velocities of spherical and ellipsoidal bubbles using a viscous correction due to Joseph and Wang [27] for the viscous pressure. Here we extend this approach to estimate bubble deformations and transient effects. As we shall see, the self-consistent bubble shapes so obtained are close to perfect oblate ellipsoids, therefore we expect this approach will yield quantitatively correct results. The practical utility of this approach is illustrated by comparing predictions of this approach with experimental results summarized earlier.

2. Formulation

The velocity field $\mathbf{u}$ of an incompressible Newtonian fluid obeys the Navier–Stokes equation [28]

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \mu \nabla^2 \mathbf{u} - \rho \mathbf{g} \tag{1}$$

and the conservation of mass condition $\nabla \cdot \mathbf{u} = 0$, where p represents the pressure, t the time and $\mathbf{g}$ the body force acting on the fluid due to gravity. The boundary condition at the surface of the bubble is given by the Young–Laplace equation for which the difference in normal stress across the bubble surface is balanced by the product of the interfacial tension, σ and the local mean curvature, κ:

$$p_{in} - p + 2\mu \frac{\partial u_n}{\partial n} = \sigma \kappa \tag{2}$$

where p_{in} is the internal pressure of the bubble, u_n is the normal component of the velocity and $\mathbf{n}$ is the unit outward normal directed into the fluid. In the above equation, $\partial/\partial n = \mathbf{n} \cdot \nabla$ represents the normal derivative. We also assume the bubble surface is fully mobile so that the tangential stress vanishes.

We employ the exact Helmholtz decomposition: $\mathbf{u} \equiv \mathbf{u_p} + \mathbf{v} = \nabla\phi + \mathbf{v}$, where $\mathbf{u}$ is written as a sum of an irrotational field, $\nabla\phi$ (with ϕ the velocity potential) and a rotational field, $\mathbf{v}$. Eq. (1) can then be recast as

$$\nabla \left\{ \rho \frac{\partial \phi}{\partial t} + \frac{1}{2} \rho |\nabla \phi|^2 + p + p_v + \rho g z \right\} = 0 \tag{3}$$

where the viscous pressure, p_v is given by

$$\nabla p_v \equiv \rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla)(\nabla \phi) + \rho (\nabla \phi \cdot \nabla) \mathbf{v} + \rho \mathbf{v} \cdot \nabla \mathbf{v} - \mu \nabla^2 \mathbf{v} \tag{4}$$

At high Reynolds numbers, the irrotational part of the velocity $\nabla\phi$ provides a uniformly valid leading order approximate solution to the velocity field so that $\partial u_n/\partial n \cong \partial^2\phi/\partial n^2$ [29]. The evolution of the potential and the position $\mathbf{X}$ of an element of the bubble