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Maximum likelihood estimator consistency for recurrent random walk in a parametric random environment with finite support

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Abstract

We consider a one-dimensional recurrent random walk in random environment (RWRE) when the environment is i.i.d. with a parametric, finitely supported distribution. Based on a single observation of the path, we provide a maximum likelihood estimation procedure of the parameters of the environment.

Unlike most of the classical maximum likelihood approach, the limit of the criterion function is in general a non degenerate random variable and convergence does not hold in probability. Not only the leading term but also the second order asymptotic is needed to fully identify the unknown parameter. We present different frameworks to illustrate these facts. We also explore the numerical performance of our estimation procedure.

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1. Introduction

Since the pioneer works of Chernov [8] and Temkin [21], random walks in random environments (RWRE) have attracted many probabilists and physicists, and the related literature in these fields has become richer and source of fine probabilistic results that the reader may find in surveys including Hughes [15] and Zeitouni [22]. Introduced originally by Chernov [8] as a model for DNA replication, RWRE were recently used by Baldazzi et al. [6,5] and Andreoletti and Diel [3] to analyze experiments on DNA unzipping, pointing the need of specific statistical procedures. The literature dealing with the statistical analysis of RWRE is far from being rich and we aim at making a contribution to the inference of parameters of the environment distribution for a one-dimensional nearest neighbor path.

1.1. The model

Let $\omega = (\omega_x)_{x\in\mathbb{Z}}$ be an independent and identically distributed collection of $(0,1)$-valued random variables with a parametric distribution η_θ. The process ω represents a random environment in which the random walk evolves. Denote by $\mathbb{P}^\theta = \eta_\theta^{\otimes\mathbb{Z}}$ the law on $(0,1)^{\mathbb{Z}}$ of ω and by $\mathbb{E}^\theta$ the expectation under this law.

For fixed environment ω, let $X = (X_t)_{t\in\mathbb{Z}_+}$ be the Markov chain on $\mathbb{Z}_+$ starting at $X_0 = 0$ and with transition probabilities $P_\omega(X_{t+1} = 1|X_t = 0) = 1$, and for $x > 0$

$$P_\omega(X_{t+1} = y|X_t = x) = \begin{cases} \omega_x & \text{if } y = x+1, \\ 1-\omega_x & \text{if } y = x-1, \\ 0 & \quad\text{otherwise.} \end{cases}$$

For simplicity, we stick to the RWRE on the positive integers reflected at 0, but our results apply for the RWRE on the integer axis as well. The symbol P_ω denotes the measure on the path space of X given ω, usually called *quenched* law. The (unconditional) law of X is given by

$$\mathbf{P}^\theta(\cdot) = \int P_\omega(\cdot)\mathrm{d}\mathbb{P}^\theta(\omega),$$

this is the so-called *annealed* law. We write E_ω and $\mathbf{E}^\theta$ for the corresponding quenched and annealed expectations, respectively.

In this work we restrict the model to environments with finite support of the form

$$\eta_\theta = \sum_{i=1}^{d} p_i \delta_{a_i}, \tag{1}$$

with d an integer, $\mathbf{p} = (p_i)_{1\le i\le d}$ a probability vector and $\mathbf{a} = (a_i)_{1\le i\le d}$ the ordered support. We further assume that $d \ge 2$ is known, and the unknown parameter is $\theta = (\mathbf{a}, \mathbf{p})$.

This framework already reveals the main features of the estimation problem and also covers some interesting applications, such as a DNA-unzipping model.

1.2. Motivating example: DNA-unzipping model

The DNA molecule is a double strand of the nucleotides base pairs. Denote one strand of the DNA chain by $(b_1, b_2, \ldots)$, where $b_x \in \{A, C, G, T\}$ is the xth base. The corresponding base on the other strand is determined by the pairing rule: A can only be paired with T while G can only be paired with C.