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Knowing By Number: Learning Math for Thinking Well

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If late modernity's central measure of cultural import is the viral YouTube video, then the Common Core mathematics standards have proved infamous indeed. Karen Lamoreaux's 2013 appearance before the Arkansas Board of Education has been viewed over three million times and is evocatively titled, "Arkansas Mother Obliterates Common Core in 4 Minutes!"[1] Lamoreaux's gripe was not with the standards per se, but rather with the method of problem solving the standards apparently promoted. Her fourth grader's class of eighteen students was presented with this conundrum: if they "counted around" by some particular number, and ended on ninety, with what number did they count around?[2] Lamoreaux points out that she would have solved it by asking herself what number times eighteen produced ninety, namely five. Two steps, and done. The students, apparently, were instead asked to draw eighteen circles, and then ninety hash marks, evenly distributed, to arrive at the answer of five per circle—a process that took a minimum of 108 steps. Surely this, Lamoreaux suggested, was decisive evidence of the curriculum's intellectual bankruptcy.

Nineteen seventy-two's version of the viral video was the "Letter to the Editor," and that autumn the Washington Post received an astoundingly high number of letters concerning an article that expressed a strikingly similar complaint. James Shackelford, PhD, a chemist with the Environmental Protection Agency, found that he did not understand the problems in his daughter's fourth grade math assignment, taken from one of the novel "new math" textbooks. Far worse, he found neither his daughter nor any of her friends knew the answer to his question, "What is eight times nine?" from memory. The Post featured a front-page story on his complaints, the upshot of which echoed Lamoreaux's: surely, Shackelford claimed, his experience was decisive evidence of the curriculum's failure.

Both episodes point to the seemingly inescapable conclusion that math class is about more than just reaching the "right" answer. You also have to get it in the "right" way. Many fields, after all, convey useful knowledge—astronomy, geography, home economics—but only a few are said to be "good for the mind," training students to face complicated problems generally. For millennia, mathematics has been thought of as the discipline that disciplines—putting the mind on the right track and training rational judgment. This assumption is still alive and well in the twenty-first century in the way most people talk about intelligence—a teenager who's a great mathematician is a genius, but a teenager who's a magnificent poet is, well, just a poet. As those applying to enter American graduate schools in English or social work quickly realize, the required Graduate Record Exam still demands mathematical competence, not knowledge of literature or psychology.

Mathematical practice is both a resource for and model of rigor, precision, proof, and certainty. In one of Plato's dialogues, Socrates shows Meno how even an uneducated slave "already knows" how to construct a specific sort of geometric figure. Socrates concludes that the slave had "true opinions on a subject without having knowledge." That is, mathematical knowledge was not from "teaching but from questioning"—geometry class does not convey knowledge but reveals true knowledge within all of us.[3] Christians, too, heralded mathematics' close relationship with reason—that God had written the "book of nature" in the language of mathematics, legible only though the application of uniquely human intelligence. The eighteenth-century mathematician Jean-Etienne Montucla declared that a well-done history of mathematics "could be looked upon as a history of the human mind, since it is in this science more than all others that man makes known the excellence of the gift of intelligence which God has given him to raise him above all other creatures."[4] And, in the nineteenth century, university positions in mathematics were doled out in part on the basis of what kind of mathematical instruction might be most helpful for young minds. When Sir William Hamilton wrote in support of Duncan Gregory's case for the University of Edinburgh Chair of Mathematics, he did so on the basis of Gregory's preference for geometry over algebra: "The mathematical process in the symbolical method [i.e., the algebraic] is like running a rail-road through a tunnelled mountain; that in the ostensive [i.e., the geometrical] like crossing the

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[1] "Arkansas Mother Obliterates Common Core in 4 Minutes!," *YouTube*, accessed April 25, 2016, https://www.youtube.com/watch?v=wZEGijN_8R0.

[2] "Counting around" means counting in multiples; counting around by seven would mean counting {7, 14, 21, . . .}.

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[3] Plato, "Meno," in *The Collected Dialogues of Plato Including the Letters*, ed. Edith Hamilton and H. Cairns, trans. W. K. C. Guthrie (Princeton, NJ: Princeton University Press, 1961): 353–84, on 370.

[4] Galileo's Assayer contains the best-known pronouncement that the "book of nature" was written in the language of mathematics: Stillman Drake, ed. and trans., *Discoveries and Opinions of Galileo* (New York, NY: Anchor Books, 1957), 238. Montucla quoted in Joan L. Richards, "Historical Mathematics in the French Eighteenth Century," *Isis* 97 (2006): 700–713, on 707. Similar assertions are made elsewhere, e.g., Lancelot Hogben, *Mathematics for the Million: A Popular Self Educator* (London: George Allen & Unwin, 1936), 34.

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mountain on foot. The former carries us, by a short and easy transit, to our destined point, but in miasma, darkness and torpidity, whereas the latter allows us to reach it only after time and trouble, but feasting us at each turn with glances of the earth and of the heavens, while we inhale health in the pleasant breeze, and gather new strength at every effort we put forth."[5] For Hamilton, as for many observers, the debate was not about which practice produced superior research, but which would promote the best form of reasoning.

Lamoreaux's complaint about the Common Core is just the latest iteration of a long history of worries that math class was failing to teach students to think in the right way. It was not simply that textbooks had factual errors or were pedagogically deficient. Rather, if mathematical truths are certain and reliable, then learning them entailed learning how to recognize certain and reliable knowledge more generally. As mathematics became a standard course of study over the last century, math educators have often encouraged this view that math education was general education. A 1923 Mathematical Association of America report on the proper aims and nature of mathematics instruction reasserted the centuries-old claim that "general mental discipline is a valid aim in education" and made the case that mathematics training might "transfer" between disciplines. At that time, the key was promoting a (non-technical) notion of mathematical functions to the expanding population of high school students, as the basis of "functional thinking" generally. In 1940, the Progressive Education Association emphasized a similarly transferable role for mathematics, no longer on the basis of functions but rather as a set of practical, specific skills for general education.[6]

Perhaps no historical example paralleled Lamoreaux's complaint so directly as did the new math reforms that prompted Shackelford's story in the Washington Post.[7] Though contemporaneous reforms also took place elsewhere at mid-century, the American reforms were in many cases the most explicitly political. The new math began as one of a number of National Science Foundation (NSF) projects to shape the nation's curriculum by engaging academic scientists to write model textbooks. Unlike the situation in many countries, American education was and is entirely decentralized, with states, local districts, and individual classroom teachers in charge of textbook selection and curricular decisions. With federal funding, the National Science Foundation provided time-and-a-half salary starting in 1958 for mathematicians to join with high school teachers and rewrite the nation's textbooks for grades nine through twelve. Eventually the School Mathematics Study Group, as the key NSF-funded initiative was called, expanded to all grades, as well as to "disadvantaged" and "slow" students. Though there were other reform efforts, only the School Mathematics Study Group had the imprimatur of official status, as it was supported by the American Mathematical Society, Mathematical Association of America, and the National Council of Teachers of Mathematics in addition to the NSF.

The NSF's curricular reform program had been initiated more than a year before Sputnik, but the launch of the Soviet satellite in the fall of 1957 meant that funding would be extraordinarily generous. In the two years after the launch, NSF spending on education projects increased over tenfold. The curricular program had widespread bipartisan support at first, with liberals pleased that money was being spent on education and conservatives happy that education funding was limited to mathematicians and scientists, rather than so-called progressive educators. Representatives from both sides of the aisle thought that model textbooks provided a mechanism for curricular change without impinging on states' and school districts' rights.

Moreover, the reforms were understood as addressing the problems of Cold War manpower shortages. For years, the NSF had published a Scientific Manpower Bulletin and fear mongers had consistently claimed (based on often spurious evidence) that the Soviet Union was producing far more scientists and engineers. As physicist Henry Smyth explained, scientists were "tools of war" to be "stockpiled" just "as we would any other essential resource."[8] Though the original mission of the Foundation was graduate research fellowships, then-director Alan Waterman soon realized that Congress would be far more willing to increase funding if pre-collegiate educational initiatives were included.

The politics of "scientific manpower" did not dictate what sort of mathematics to teach, only that it should be taught more. Many mathematicians involved with the School Mathematics Study Group, however, did have specific ideas about how math should be taught. They thought the discipline had for too long been portrayed as a "dead and completed subject that was embalmed between the covers of a textbook sometime after Sir Isaac Newton."[9] Moreover, teaching math as a "dead" subject also often meant teaching it as a set of facts to be memorized. This was not just stultifying, but also meant learning math in a way that could easily have been described as authoritarian. How might scientific education aid in the defeat of the Soviets if it were taught in such a way as to promote blind deference to authority? It was in this context that Howard Fehr, a School Mathematics Study Group supporter and one-time head of the National Council of Teachers of Mathematics, explained that "It is essential for the student of mathematics to have acquired the faculty of being able, by his own wit, to learn more mathematics, to solve new problems, to adapt his past knowledge to new knowledge and new points of view; and, above all, it is essential to him to have been liberated from the shackles of authority."[10] Likewise, a School Mathematics Study Group–allied reform effort

[5] George Davie, *The Democratic Intellect: Scotland and Her Universities in the Nineteenth Century*, 2nd ed. (Edinburgh: Edinburgh University Press, 1964), 127.

[6] The National Committee on Mathematical Requirements, *The Reorganization of Mathematics in Secondary Education* (Oberlin, OH: Mathematical Association of America, 1923), 8, 72, 90; Commission on the Secondary School Curriculum of the Progressive Education Association. *Mathematics in General Education: A Report of the Committee on the Function of Mathematics in General Education* (New York, NY: 1940), 11–12.

[7] The material of this section is discussed in much greater detail in Christopher J. Phillips, *The New Math: A Political History* (Chicago: University of Chicago Press, 2015).

[8] Henry D. Smyth, "The Stockpiling and Rationing of Scientific Manpower," *Bulletin of the Atomic Scientists* 7, no. 2 (1951): 38–42, 64. For the spurious claims of "shortages," see David Kaiser, "The Physics of Spin: Sputnik Politics and American Physicists in the 1950s," *Social Research* 73 (2006): 1225–52.

[9] School Mathematics Study Group, *Mathematics for Junior High School: Student's Text*, vol. 2 (New Haven, CT: Yale University Press, 1961), 545.

[10] Howard F. Fehr, "Modern Mathematics and Good Pedagogy," *Arithmetic Teacher* 10 (1963), 402–11, on 403–4.