Arthur Cayley, Robert Harley and the quintic equation: newly discovered letters 1859–1863

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Abstract

Beginning in the 1840's, Arthur Cayley (1821–1895) led a vast invariant theory programme in algebra. After learning of results of James Cockle (1819–1895) and Robert Harley (1828–1910), he applied the techniques of invariant theory to the calculation of resolvents of quintic equations. Letters recently discovered reveal the priorities of Cayley and Harley with respect to the quintic, an approach which was at variance with that *via* the theory of groups. As another recently discovered manuscript reveals, Cayley returned to this subject in his final days.
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Résumé

À partir de la période de 1840, Arthur Cayley (1821–1895) a mené un vaste programme de la théorie des invariants en algèbre. Après avoir appris les résultats de James Cockle (1819–1895) et Robert Harley (1828–1910), il a appliqué les techniques de la théorie des invariants pour le calcul de résolvantes d'équations du cinquième degré. Lettres récemment découvertes révèlent les priorités de Cayley et Harley par rapport aux équations du cinquième degré, une approche differente que par la théorie des groupes. Comme un autre manuscrit récemment découvert révèle, Cayley est revenu à ce sujet dans ses derniers jours.
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1. Introduction

When Arthur Cayley's wife died in 1923 his correspondence, assiduously collected for over half a century, was mostly destroyed. Aside from some letters written to and from James Joseph Sylvester, held in an archive at St. John's College Cambridge, few written to Cayley survive, and those written by Cayley are scattered. This background makes the coming to light of a batch of letters written by Cayley to the little known mathematician Robert A. Harley especially valuable, even though the period of this batch is brief – a mere four years. Robert Broadhead Honeyman, a well-known manuscript collector, donated a large collection of material to the Lehigh University library in 1958. Much of this collection lay unexamined by scholars for decades, but in the past few years it has been digitized and made available on the library's website [Lehigh]. This collection includes a set of 40 letters from Arthur Cayley to Robert Harley, written between 1859 and 1863. It also includes a manuscript, drafts of a *Memoir on the Quintic Equation*, on which Cayley was working at the time of his death in 1895 which, as will appear, is related in subject matter to some of the letters. We report on this material, which sheds light on Cayley's working relationship with Harley, and by extension on his relationship with coworkers, and also on Cayley's lifelong fascination with the quintic equation. We note that the letters are half of a correspondence, the other half of which has not been preserved. (Actually, it is slightly less than half, as some of the letters refer to material sent from Cayley to Harley that is not part of this collection and apparently has not been preserved.) The letters are all simply addressed "Dear Sir", but we have been able to identify the recipient by internal evidence, as will be clear below. Also, the draft *Memoir* is undated, but again, internal evidence allows us to date it, as will also be clear below.

Cayley's papers sometimes present the results of extensive calculations without providing much context. These letters make clear that many of the papers from the period belong to a large research programme concerning the quintic equation which encompassed other English mathematicians with whom Cayley interacted. We begin this paper by giving the personal background to the correspondence, then give an introduction to the mathematics discussed in the correspondence and in the draft *Memoir*. In the heart of the paper, we consider the Cayley–Harley correspondence, followed by a brief summary of the contents of the *Memoir* and a conclusion.

2. The correspondents and the occasion of their correspondence

The letters in the Lehigh collection begin when Cayley was 38 years and Harley 31 years old. During the 1850s Cayley was firmly established as England's leading pure mathematician, but he was unsuccessful in finding an academic position until June 1863, (just after the end of the correspondence being discussed here) when he became the first Sadleirian professor of Pure Mathematics at Cambridge. Instead, he was practicing law, based at Stone Buildings Lincoln's Inn in London. His professional activity lay in drafting legal documents, an activity which hardly demanded his appearance in court and to a large extent allowed him to regulate his own time. (For more biographical details, see [Forsyth, 1895], [Crilly, 2006].) During the same period, Harley was a Congregationalist minister at Brighouse in Yorkshire. In 1844, at the age of 16, he had become interested in problems in "The Lady's and Gentleman's Diary" (the magazine in which the famous Kirkman's schoolgirls problem appeared in 1850), which brought him into contact with James Cockle, then well-known to English mathematicians. Harley was very much interested in, and influenced by, Cockle's work, and they became life-long friends. Indeed, upon Cockle's death in 1895 Harley wrote his obituary for the *Proceedings of the Royal Society* [Corley and Crilly, 2004]. In the 1850s Cockle and Harley joined forces on the quintic problem, and between 1858 and 1862 they were in almost daily communication. Harley also became a good friend, and follower, of George Boole. Harley's early mathematical work was concentrated in algebra, particularly in the theory of equations and invariant theory, which were closely linked, but later on he became interested in differential equations, particularly in their connection with