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The directionality of distinctively mathematical explanations

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ABSTRACT

In "What Makes a Scientific Explanation Distinctively Mathematical?" (2013b), Lange uses several compelling examples to argue that certain explanations for natural phenomena appeal primarily to mathematical, rather than natural, facts. In such explanations, the core explanatory facts are modally stronger than facts about causation, regularity, and other natural relations. We show that Lange's account of distinctively mathematical explanation is flawed in that it fails to account for the implicit directionality in each of his examples. This inadequacy is remediable in each case by appeal to ontic facts that account for why the explanation is acceptable in one direction and unacceptable in the other direction. The mathematics involved in these examples cannot play this crucial normative role. While Lange's examples fail to demonstrate the existence of distinctively mathematical explanations, they help to emphasize that many superficially natural scientific explanations rely for their explanatory force on relations of stronger-than-natural necessity. These are not opposing kinds of scientific explanations; they are different aspects of scientific explanation.

1. Introduction

In "What Makes a Scientific Explanation Distinctively Mathematical?" (2013b), Lange uses several compelling examples to argue that certain natural phenomena are best explained by appeal to mathematical, rather than natural, facts. In distinctively mathematical explanations, the core explanatory facts are modally stronger than facts about, e.g., statistical relevance, causation, or natural law. A distinctively mathematical explanation might describe causes, Lange allows, but its explanatory force derives ultimately from appeal to facts that are 'more necessary' than causal laws. Lange advances this thesis to argue for the importance of a purely modal view of explanation (a view that emphasizes necessities, possibilities, and impossibilities, showing that an event had to or could not have happened) in contrast to the widely discussed ontic view (a view that associates explanation with describing the relevant natural facts, e.g., about how the event was caused or how its underlying mechanisms work).[1]

Lange operates with a narrower understanding of the ontic conception. He describes it as the view that all explanations are causal. He cites Salmon, who claimed that, "To give scientific explanations is to show how events and statistical regularities fit into the causal structure of the world" (Salmon, 1984)[2] and "To understand why certain things happen, we need to see how they are produced by these mechanisms [processes, interactions, laws]" (Salmon, 1984). He also cites Lewis ("Here is my main thesis: to explain an event is to provide some information about its causal history"; 1986) and Sober ("The explanation of an event describes

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[1] There is a growing body of literature on mathematical explanation (Baker, 2005; Baker & Colyvan, 2011; Huneman, 2010; Pincock, 2011). We focus on Lange because his examples have become canonical and because his commitments are so explicitly formulated. We suspect that the directionality problem will arise in these other papers as well, but these authors are mostly concerned with indispensability and the ontology of mathematics, a topic that we (like Lange) hope to sidestep to focus on explanation alone. See Craver (2016) for a discussion of directionality problems in network explanation. Andersen's (forthcoming) response to Lange is complementary to ours, fleshing out a point about explananda at which we only gesture in the conclusion. Our main focus is directionality.

[2] See the passages quoted in Povich (forthcoming) for evidence that Salmon did not think the ontic conception was strictly causal. As we note, Lange's conception of the ontic conception is narrower than one might allow. The primary aim of the ontic conception is to insist that whether X explains Y is an objective matter of (natural) fact.

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the 'causal structure' in which it is embedded"; 1984).[3] In contrast to Lange, we adopt a more inclusive understanding of the ontic that embraces any natural regularity (Salmon, 1984; Craver, 2007, 2014; Povich, forthcoming), e.g., statistical relevance (Salmon, 1971), natural laws (Hempel, 1965), or contingent compositional relations might also figure fundamentally in explanation. This point will become crucial below, given that the ontic relations that explain the directionality of some explanations are not specifically causal relations; but they are ontic in this wider sense.[4] Lange's arguments should, however, work equally well against this broader understanding of the ontic conception, given that he uses the examples to show that some explanations of natural facts depend fundamentally on relations of necessity that are stronger than mere natural necessity.

We argue that Lange's account of distinctively mathematical explanation is flawed. Specifically, it fails to account for the directionality implicit in his examples of distinctively mathematical explanation. This failure threatens Lange's argument because it shows that his examples do not, in fact, derive their explanatory force from mathematical relations alone (independent of ontic considerations). The inadequacy is in each case easily remediable by appeal to ontic facts that account for why the explanation is acceptable in one direction and unacceptable in the other. That is, Lange's exemplars of distinctively mathematical explanation appear to require for their adequacy appeal to natural, ontic facts about, e.g., causation, constitution, and regularity. More positively, we suggest that all mechanistic explanations are constrained, and so partly constituted, by both ontic and modal facts. Rather than seeing an opposition between distinctively mathematical explanations and causal (or more broadly ontic) explanations, Lange's examples, as we reinterpret them, direct us to understand how these distinct aspects of explanation, these distinct sources of explanatory power, intermingle and interact with one another in most scientific explanations.

2. Lange's account of distinctively mathematical explanation

Lange's goal is to show "how distinctively mathematical explanations work" by revealing the "source of their explanatory power" (486). He accepts as a basic constraint on his account that it should "fit scientific practice," that is, that it should judge as "explanatory only hypotheses that would (if true) constitute genuine scientific explanations" (486). In short, the account should not contradict too many scientific common-sense judgments about whether an explanation is good or bad. Lange's goal and his guiding constraint are conceptually related: to identify the source of an explanation's power requires identifying the key features that sort acceptable explanations from unacceptable explanations of that type. In causal explanations, for example, much of the explanatory power comes from knowledge of the causal relations among components in a mechanism. Bad causal explanations of this kind fail when they misrepresent the relevant causal structure (in ways that matter). In distinctively mathematical explanations, on Lange's view, the explanatory force comes from mathematical relations that are 'more necessary' than mere causal or correlational regularities.

Given this set-up, Lange's account of the explanatory force of distinctively mathematical explanations can be undermined by examples that fit Lange's account but that would be rejected as bad explanations as a matter of scientific common-sense. The account would fail to identify fully the explanatory force in such explanations and so would fail to account for the norms governing such explanations.

Lange does not address the canonical form of distinctively mathematical explanations. However, his examples are readily reconstructed as arguments in which a description of an explanandum phenomenon follows from an empirical premise (EP) describing the relevant natural facts, and a mathematical premise (MP) describing one or more more-than-merely-naturally-necessary facts. To begin with Lange's simplest example:

Strawberries: Why can't Mary divide her strawberries among her three kids?[5] Because she has 23 strawberries, and 23 is not divisible by three.

This explanation can be reconstructed as an argument:

1. Mary has 23 strawberries (EP)
2. 23 is indivisible by 3 (MP)

C. Mary can't divide the strawberries equally among her three kids.[6]

We would have to tighten the bolts to make the argument valid (e.g., no cutting of strawberries is allowed), but the general idea is clear enough. The empirical premise works by describing the natural features of a system. They specify, for example, the relevant magnitudes (Mary starts with 23 strawberries), and the causal or otherwise relevant dependencies among them. All distinctively mathematical explanations of natural phenomena require at least some empirical premises to show how the mathematics will be applied and to specify the natural (empirically discovered) constraints under which the mathematical premises do their work. The question is whether those mathematical premises are supplying the bulk of the 'force' of the explanation, as appears to be the case in Strawberries.[7]

Lange's other examples can similarly be reconstructed as arguments mixing empirical and mathematical premises:

> **Trefoil Knot**: Why can't Terry untie his shoes? Because Terry has a trefoil knot in his shoelace (EP). The trefoil knot is not isotopic to the unknot in three dimensions (EP), and only knots isotopic to the unknot in three dimensions can be untied (MP) (489).

[3] One can believe that mechanistic explanation is important without believing that all explanations are causal or mechanical. We show why $C = 2\pi r$ without describing mechanisms. We explain why Obama can sign treaties without describing causes. Explanations in epistemology, logic, and metaphysics often work without describing causes. The question here is not whether one should be a pluralist about explanation but about whether Lange's account of distinctively mathematical explanation is complete and whether his contrast with the ontic conception is substantiated by his examples.

[4] For purposes of focus, we leave aside the question of whether the existence of distinctively mathematical explanations in fact commits one to the denial of the ontic conception or even to the idea that there is a modal form of explanation independent of ontic considerations. The fact that mathematics is important to explanation doesn't necessarily commit one to the idea that the modal conception has a role to play independently of ontic considerations absent further commitments about the relationship between mathematics and ontology. Like Lange, we remain silent on the ontology of mathematics (492).

[5] Or "Why didn't she on some particular occasion?" or "Why didn't or couldn't anyone ever?" Lange intends all these explananda to be explained by the same explanans; a similar multiplicity of explananda can be generated for the examples below.

[6] This example is reconstructed as a sketch of a deductive argument, but distinctively mathematical explanations might be inductive. For example, one might explain why fair dice will most likely not roll a string of ten consecutive double-sixes on mathematical grounds, using logical probability and some math.

[7] Lange might object to the inclusion of the empirical premise in this formulation. Instead, he might treat the empirical premise as a presupposition of the why question: "Why can't Mom divide her 23 strawberries among her three kids?" Answer: "Because 23 is indivisible by 3." In what follows, all of our examples can be so translated without affecting the principled incompleteness in the cases, but this reformulation comes at considerable cost to the clarity with which the incompleteness can be displayed (see Section 4).