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Comparing dualities and gauge symmetries

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ABSTRACT

We discuss some aspects of the relation between dualities and gauge symmetries. Both of these ideas are of course multi-faceted, and we confine ourselves to making two points. Both points are about dualities in string theory, and both have the 'flavour' that two dual theories are 'closer in content' than you might think. For both points, we adopt a simple conception of a duality as an 'isomorphism' between theories: more precisely, as appropriate bijections between the two theories' sets of states and sets of quantities.

The first point (Section 3) is that this conception of duality meshes with two dual theories being 'gauge related' in the general philosophical sense of being physically equivalent. For a string duality, such as T-duality and gauge/gravity duality, this means taking such features as the radius of a compact dimension, and the dimensionality of spacetime, to be 'gauge'.

The second point (Sections 4–6) is much more specific. We give a result about gauge/gravity duality that shows its relation to gauge symmetries (in the physical sense of symmetry transformations that are spacetime-dependent) to be subtler than you might expect. For gauge theories, you might expect that the duality bijections relate only gauge-invariant quantities and states, in the sense that gauge symmetries in one theory will be unrelated to any symmetries in the other theory. This may be so in general; and indeed, it is suggested by discussions of Polchinski and Horowitz. But we show that in gauge/gravity duality, each of a certain class of gauge symmetries in the gravity/bulk theory, viz. diffeomorphisms, *is* related by the duality to a position-dependent symmetry of the gauge/boundary theory.

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1. Introduction

The ideas of duality and gauge symmetry are among the most prominent in modern physics; and they are both related to the philosophical questions of how best to define (a) physical theories, and (b) the relation of theoretical equivalence (of 'saying the same thing') between theories. In this paper, we will make two main points about how duality and gauge symmetry are connected.

Both points are about dualities in string theory, and both have the 'flavour' that two dual theories are 'closer in content' than you might think. For both points, we adopt a simple conception of a duality as an 'isomorphism' between theories. In Section 3, we state this conception, and briefly relate it to the philosophical questions (a) and (b). In short, we take a theory to be given by a triple comprising a set of states, a set of quantities and a dynamics; so that a duality is an appropriate 'structure-preserving' map between such triples. This discussion will be enough to establish our first point. Namely: dual theories can indeed 'say the same thing in different words'—which is reminiscent of gauge symmetries.

Our second point (Sections 4–6) is much more specific. We give a result about a specific (complex and fascinating!) duality in string theory, gauge-gravity duality: which we introduce in Section 4, using Section 3's conception of duality. We state this result in Section 5. (Details, and the proof, are in De Haro, 2016.) It says, roughly speaking, that each of an important class of gauge symmetries in one of the dual theories (a gravity theory defined on a bulk volume) is mapped by the duality to a gauge symmetry of the other theory (a conformal field theory defined on the boundary of the bulk volume). This is worth stressing since some discussions suggest that all the gauge symmetries in the bulk

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theory will not map across to the boundary theory, but instead be 'invisible' to it. As we will see, this result also prompts a comparison with the hole argument. It also relates to recent discussion of the empirical significance of gauge symmetries—a topic we take up in Section 6.

To set the stage for these points, especially the second, Section 2 will describe the basic similarity between the ideas of duality and gauge symmetry: that they both concern 'saying the same thing, in different words'.

2. 'Saying the same thing, in different words'

We have already sketched the idea of a *duality* as a 'structure-preserving' map between two physical theories, or between two formulations of a single theory. Typically, there is a map that relates the theories—states, quantities and dynamics—in a 'structure-preserving way'.

As to *gauge symmetry*, we need to distinguish[1]: (i) a general philosophical meaning, and (ii) a specific physical meaning; and similarly, for cognate terms like 'gauge theory', 'gauge-dependent' and 'gauge-invariant'. As follows:

(i) (Redundant): If a physical theory's formulation is redundant (i.e. roughly: it uses more variables than the number of degrees of freedom of the system being described), one can often think of this in terms of an equivalence relation, 'physical equivalence', on its states; so that gauge-invariant quantities are constant on an equivalence class and gauge-symmetries are maps leaving each class (called a 'gauge-orbit') invariant. Leibniz's criticism of Newtonian mechanics provides a putative example: he believed that shifting the entire material contents of the universe by one meter must be regarded as changing only its description, and not its physical state.

(ii) (Local): If a physical theory has a symmetry (i.e. roughly, a transformation of its variables that preserves its Lagrangian)[2] that transforms some variables in a way dependent on spacetime position (and is thus 'local') then this symmetry is called 'gauge'. In the context of Yang–Mills theory, these variables are 'internal', whereas in the context of General Relativity, they are spacetime variables—both types of examples will occur in Sections 4f.

Although (Local) is often a special case of (Redundant), it will be important to us that this is not always so. For we will be concerned with (Local) gauge symmetries (specifically: diffeomorphisms) which are asymptotically non-trivial (i.e. do not tend to the identity at spacelike infinity), and which can thus change the state of a system relative to its environment.[3]

These sketches are enough to suggest that for any theory, or for any theory and its duals, duality and gauge-symmetry are likely to be related. The obvious suggestion to make is that the differences between two dual theories will be like the differences between two formulations of a gauge theory: they 'say the same thing, despite their differences'. Indeed, for several notable dualities, this is the consensus among physicists. But the details vary from case to case, and can be a subtle matter: much depends on how we interpret the vague phrase 'will be like'.

The subtleties arise, in part, from the fact that physicists tend to call a correspondence or map that 'preserves what is said' a "duality", only when it is: (1) striking and-or (2) useful, in one way or another—and in a way that gauge-symmetries typically are *not*. It is worth spelling out these desiderata a little. It will show how we should interpret the phrase 'will be like'; that is, it will enable us to disambiguate and assess the suggestion.

(1) *Striking*: One main way the correspondence can be *striking* is that the two 'sayings' (i.e. the two formulations) are very disparate. The most striking examples of this occur in string theory (which will be our focus). Thus in S-duality, the two formulations differ about the electric or the magnetic nature of the charges. In T-duality, the two formulations differ about the radius of a compact dimension of space: where one says it is r, the other says it is $1/r$. And in gauge-gravity duality (on which we focus from Section 4), the formulations differ about the dimensionality of spacetime (and of course, much else!): where the 'bulk' formulation says it is $d+1$ (say 5), the other 'boundary' formulation says it is d (say 4).

Agreed: such a difference—of electric *vs.* magnetic charge, or the radius or dimensionality of spacetime—seems so marked that you might well doubt that the formulations are 'saying the same thing'. You might well say instead that they contradict each other; so that the example is best taken as a case of under-determination (of theory by all possible data), not as any kind of theoretical equivalence. This is a reasonable reaction: for example, McKenzie (2016, Section 4) argues for this view, as regards S-duality. More generally, we agree that the relations between dualities and theoretical equivalence, especially for string theory, are by no means settled, as several recent philosophical discussions witness. (Examples for our main case, gauge/gravity duality, which also discuss claims that one side of the duality (usually the gravity side) is 'emergent' from the other, include: De Haro (2016a, especially Section 3.2), Dieks, van Dongen, & de Haro (2016, especially Section 3.3), Rickles (2012,especially Section 5), and Teh (2013, especially Sections 3 & 4).

But in this paper, we will not need to decide these issues, nor even the best interpretation of gauge/gravity duality's case of dimensionality; though we will briefly return to the issues in Sections 3 and 4. For the moment, it is enough to report the consensus in string theory: that at least some of these striking dualities *do* relate different formulations of a single theory. As the physics jargon has it: they 'describe the same physics'; meaning of course not just observational, but also theoretical, equivalence. And several philosophical commentators endorse this consensus, including for our case of gauge/gravity duality; e.g. De Haro (2016a, Section 2.4.1), Dieks et al. (2016, Section 3.3.2), Huggett (2016, Sections 2.1 & 2.3), Rickles (2011, Sections 2.3 & 5.3), Rickles (2012, Section 6), Rickles (2016), and Matsubara (2013, Section 6).

Besides, some string theorists go further. They take the ongoing search for an M theory to be the search for a formulation which will relate to the present formulations, on the two sides of such dualities, in much the way a gauge-invariant formulation of a gauge theory relates to different choices of gauge.[4]

(2) *Useful*: One main way the correspondence can be *useful* is if it relates a regime where problems are difficult to solve (say because couplings are strong) to one where they are easy to solve

[1] We should note that many theoretical physicists would take 'gauge symmetries' to mean 'small and asymptotically trivial symmetries', at least in the case of a subsystem. However, our labeling will serve as a helpful clarification, since 'gauge' is a term about which there is still much confusion.

[2] There are two kinds of subtlety, that come up even in the classical context: (i) the relation between preserving the Lagrangian (called a 'variational symmetry') and preserving the equations of motion (a 'dynamical symmetry'); (ii) the need in Hamiltonian mechanics to preserve the symplectic form, as well as the Hamiltonian. For an exposition of (i) and (ii), cf. e.g. Butterfield (2006, especially Sections 3.2, 3.4, 5.3, and 6.5). Suffice it to say here that, for the quantum case we will be concerned with in Section 3f, which will be treated in a Lagrangian i.e. path-integral framework: a symmetry must preserve the Lagrangian and the measure.

[3] Cf. the discussion in Greaves & Wallace (2014) and Teh (2016); cf. also Section 6.

[4] Indeed, the discovery of T-duality was one of the factors that prompted the idea of M theory: see Witten (1995).