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Making sense of absolute measurement: James Clerk Maxwell, William Thomson, Fleeming Jenkin, and the invention of the dimensional formula

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A R T I C L E I N F O

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A B S T R A C T

During the 1860s, the Committee on Electrical Standards convened by the British Association for the Advancement of Science (BAAS) attempted to articulate, refine, and realize a system of absolute electrical measurement. I describe how this context led to the invention of the dimensional formula by James Clerk Maxwell and subsequently shaped its interpretation, in particular through the attempts of William Thomson and Fleeming Jenkin to make absolute electrical measurement intelligible to telegraph engineers. I identify unit conversion as the canonical purpose for dimensional formulae during the remainder of the nineteenth century and go on to explain how an operational interpretation was developed by the French physicist Gabriel Lippmann. The focus on the dimensional formula reveals how various conceptual, theoretical, and material aspects of absolute electrical measurement were taken up or resisted in experimental physics, telegraphic engineering, and electrical practice more broadly, which leads to the conclusion that the integration of electrical theory and telegraphic practice was far harder to achieve and maintain than historians have previously thought. This ultimately left a confusing legacy of dimensional concepts and practices in physics.

1. Introduction

During the 1860s, the Committee on Electrical Standards convened by the British Association for the Advancement of Science (BAAS) attempted to articulate, refine, and realize a system of absolute electrical measurement. At stake was a potential alliance between mathematical physics and electrical practice. This paper describes how the Committee's apparent success depended upon making the absolute measurement of electrical resistance intelligible to telegraph engineers as well as upon devising a straightforward procedure for unit conversion. It identifies the invention and subsequent interpretation of the dimensional formula as central to both challenges, and therefore focuses upon the three key figures on the Committee—its de facto leader, William Thomson, the secretary, Fleeming Jenkin, and his collaborator, James Clerk Maxwell—who contributed to this process.

Our trio's efforts merit detailed analysis for three reasons. First, they demonstrate that the harmony of theory and practice striven for by Thomson was much harder to achieve and maintain that historians have previously thought. The conceptual, theoretical, and material aspects of absolute measurement spread unevenly and with difficulty into experimental physics, telegraphic engineering, and electrical practice. Second, Maxwell's invention of the dimensional formula initiated the incorporation of dimensional concepts and reasoning into the mainstream of theoretical and experimental physics; *that this actually happened* is not yet established, let alone understood. And third, this process left a legacy of stubborn interpretive difficulties that might be resolved or reframed via knowledge of their historical origins.[1] The historian John Roche has called this approach 'critical physics'.

In his book *The Mathematics of Measurement*, Roche recognized that Maxwell's dimensional nomenclature seemed somehow to represent, whether for Maxwell or his followers, absolute units, measurement protocols, and conversion factors all at once (Roche,

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[1] The reputation of dimensional analysis for inscrutability has endured despite the courageous attempts of twentieth-century physicist-philosophers to impose a consistent interpretation upon it. Two canonical works are Bridgman (1922), esp. 1–35 and Ellis (1966), esp. 111–51.

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2000, 203–4). Viewed from the perspective of critical physics, this paper offers a historical deconstruction of these interpretive stances. Maxwell, in an appendix to the Committee's second report co-authored with Jenkin, devised dimensional formulae as mathematical tools for unit conversion, which eventually led him, as we shall explain, to treat them as analytic forms for absolute units. With the aid of several idealized ‘illustrations’ of the absolute measurement of resistance, Thomson drew highly localized operational inferences from the dimensional formulae for resistance as a way of making absolute units of resistance intelligible, which Jenkin subsequently promoted.

Thomson's reasoning was refined and generalized in the 1880s by the French physicist Gabriel Lippmann, whose case is included here in order to illustrate the nature of the historical and conceptual processes involved in grafting supplementary uses and meanings upon dimensional formulae beyond those originally envisaged by Maxwell. Contrary to widely held belief, the findings of this paper strongly imply that neither Maxwell nor Thomson considered dimensions to reveal ‘the inner [dynamical] nature of a physical quantity’ (Roche, 2000, 202; de Clark 2010, 97), a position that the philosopher Salvo d'Agostino has advocated forcibly and unpersuasively (d'Agostino, 2000, 54–6, 64–8). We shall reinterpret the evidence brought forward so far to favor either the absolute unit or operational views outlined above. Although they all took absolute metrology to legitimate various programs of dynamical reduction, ‘dynamical reductionist’ stances towards dimensional formulae were more characteristic of Maxwell's followers than Maxwell himself.

2. Absolute measurement and the BAAS Committee on Electrical Standards

The origins of the Committee on Electrical Standards convened by the British Association for the Advancement of Science (BAAS) lay in the increasing recognition of the importance of accurate measurement to the emerging technology of submarine telegraphy. This depended heavily on the creation and distribution of uniform standards, in particular for electrical resistance. In 1861, William Thomson, and the telegraph engineers Sir Charles Bright and Latimer Clark, called for the creation of a BAAS Committee to define an interconnected system of electrical units and to construct a standard of electrical resistance. This remit was soon expanded to electrical standards more generally. Steered by Thomson's commitment to the unity of theory and practice, the Committee articulated an absolute system of dynamical and electrical units and, starting with resistance, strove to represent these units materially (Hunt, 1994, 52–60; Hunt, 2010, 84–93, 104–7; Smith, 1998, 276–8).

At this time, absolute measurement would have been familiar to only a handful of mathematically-trained natural philosophers and not at all to telegraph engineers (see §4). To understand the challenges of intelligibility it posed, we first need to explain what the Committee meant by the term and how it defined absolute electrical units. Although there are already several detailed historical accounts, none of them identify unit conversion between absolute units based on the Imperial and metric systems as a key issue that Maxwell and Jenkin addressed by inventing the dimensional formula (Smith & Wise, 1989, 684–94; Smith, 1998, 268–70, 280–3; Lagerstrom, 1992, 9–10, 24–7). We bring this into focus in this section by examining more closely the Committee's choice of fundamental unit types (length, mass, and time) and tokens (metre, gram, and second) in the context of radical political moves towards metrication during the 1860s.

Since ‘the only information on the subject now extant is scattered in detached papers by Weber, Thomson, Helmholtz, and others, requiring considerable labor to collect and understand’, the Committee decided to provide ‘a full explanation of the meaning of absolute measurement, and of the principles by which absolute electrical units are determined’ (Second Report, 1864 [1863], 112).[2] This accounted for the largely didactic orientation of the Committee's Second Report of 1863, the main body of which was likely written by Thomson (Thompson, 1910, v. 1, 419; Smith & Wise, 1989, 688). Maxwell and Jenkin took on much of the responsibility in their substantive appendix (see §2 below) entitled ‘On the elementary relations between electrical measurements’.

Implicit in the Second Report are two related criteria of absolute measurement. First, the reduction of one type of unit to another: in other words, measurement ‘made by reference to certain fundamental units of another kind treated as postulates’ as opposed to ‘a simple comparison with an arbitrary quantity of the same kind as that measured’. It gave the foot-pound as an example. If ‘the power of an engine’ were expressed in foot-pounds, then the measure referred not to ‘another source of power, such as a horse or man’, but ‘the units of weight and length simply’ (Second Report, 1864 [1863], 112).[3] Obviously measurements of the quantities chosen as fundamental are made in terms of units of the same kind and are hence comparative, but the Report does not consider whether they are also absolute.

The second criterion governed the relations between the units. The Committee explained that ‘the word absolute is intended to convey the idea that the natural connexion between one kind of magnitude and another has been attended to, and that all the units form part of a coherent system.’ (Second Report, 1864 [1863], 112) By ‘natural connexion’ or ‘natural relation’ it meant geometrical relations or empirical laws in algebraic form. For these to define a ‘coherent system’ of derived units, they had to be free from ‘useless coefficients’ or conversion factors (Second Report, 1864 [1863], 113). Thomson regarded these as a source of potential error and unnecessary expenditure of mental labor. Unit volume, for instance, had to be defined as the cube of the unit length. If this were the foot, then the unit of volume would be the cubic foot. Any other unit would necessitate the introduction of a coefficient: to obtain the numerical volume in Imperial gallons, say, would require multiplication by 6.25 (Second Report, 1864 [1863], 113). Similarly, in a manuscript draft, Maxwell likewise described how the square foot and cubic foot are ‘natural’ units of surface and volume respectively if the foot is chosen as a fundamental unit: ‘we are all familiar with the ease with which from linear dimensions we calculate superficial and cubical contents without the introduction of any coefficient’ (Maxwell MS).

Dynamical units were defined in terms of the fundamental units by following the same procedure. According to the relations given in Table 1, unit velocity would be made equal to the velocity for which unit length would be traversed in unit time, and unit force to the force that would produce ‘the unit velocity in the unit of mass when it has acted on it for the unit of time. This force acting through unit of space performs the absolute unit of work.’ (Second Report, 1864 [1863], 112) Hence the mathematical form of the relations and the absolute system of units were interdependent. The former would require coefficients to be valid for non-absolute units, whereas the latter depended upon the mathematical relations for their definition.

Electrical (and magnetic) units could then be defined in terms of forces produced and work performed; hence their sizes could

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[2] The reports are dated here according to their year of publication in the *Reports of the British Association for the Advancement of Science.* The year in brackets (the year before) is the year of the BAAS meeting at which they were presented.

[3] Here the report does not appear to differentiate clearly between work and power, or rate of performance of work, unlike Thomson & Tait, 1867, §§238–41, 176–8, §268, 187–8.