Engineering Analysis with Boundary Elements 37 (2013) 116–127

Contents lists available at SciVerse ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Three-dimensional unsteady thermal stress analysis by triple-reciprocity boundary element method

Yoshihiro Ochiai [a,*], Vladimir Sladek [b], Jan Sladek [b]

[a] Department of Mechanical Engineering, Kinki University, 3-4-1 Kowakae, Higashi-Osaka, Osaka 577-8502, Japan
[b] Institute of Construction and Architecture, Slovak Academy of Science, 845 03 Bratislava, Slovakia

ARTICLE INFO

Article history:
Received 14 March 2012
Accepted 1 September 2012
Available online 4 October 2012

Keywords:
Boundary element method
Heat conduction
Meshless method
Thermal stress

ABSTRACT

The conventional boundary element method (BEM) requires a domain integral in unsteady thermal stress analysis with heat generation and/or an initial temperature distribution. In this paper, it is shown that the three-dimensional unsteady thermal stress problem can be solved effectively using the triple-reciprocity boundary element method without internal cells. In this method, the distributions of heat generation and initial temperature are interpolated using integral equations and higher order time-dependent fundamental solutions. A new computer program was developed and applied for solving several test problems.

1. Introduction

The unsteady thermal stress problems cannot be solved easily, without using internal cells, by the conventional boundary element method (BEM), in general. Only special cases of problems, such as unsteady thermal stress problems with heat generation and initial temperature distributions being given by harmonic functions, can be solved by the standard BEM without the need for internal cells. When an analysis of thermal stress under arbitrary heat generation or a non-uniform initial temperature distribution within the domain is carried out by the BEM, a domain integral is involved in general [1,2]. However, by including the domain integral, the merit of BEM is lost, since the unknowns are not localized on the boundary alone like in pure BEM. Thus, several other methods have been considered. Nowak and Neves proposed a multiple-reciprocity method [3,4]. Tanaka et al. have proposed a dual-reciprocity BEM for transient heat conduction problems [5], and local integral equations have been proposed for unsteady heat conduction problems [6,7]. However, these methods do not employ a time-dependent fundamental solution, which can improve accuracy of numerical results. A Laplace transformation can remove the time dependence of the problem, however, it is not suitable under complicated time-dependent boundary conditions. Moreover, the Laplace transformation method requires internal cells for the initial temperature distribution and finally the numerical inverse transformation.

Recently, the efficient treatment of domain integrals has been proposed by the triple-reciprocity BEM or improved multi-reciprocity BEM for steady heat conduction, steady thermal stress and elastoplastic problems [8–10,20]. The triple-reciprocity BEM for two-dimensional heat conduction and thermal stress analysis for an unsteady state has also been proposed [11–13]. In this paper, the triple-reciprocity BEM is developed for three-dimensional unsteady heat conduction and quasi-static thermoelasticity problems. In this method, the heat generation and initial temperature distributions are interpolated using the boundary integral equations. Since the domain integrals are eliminated, no internal cells are required in the present triple-reciprocity method and the time-dependent fundamental solutions are employed. All the higher order fundamental solutions and/or corresponding integral kernels are derived in closed form including the time integrations. Besides the solution of thermoelastic initial-boundary value problems, the integral representation is also developed for post-processing computation of stresses at interior points. A new computer program was developed and applied for solving several test problems.

2. Governing equations

In the theory of thermal stresses the temperature field is not influenced by mechanical fields (such as displacements and stresses) though the latter are affected by temperature gradients. If we are interested in elastic deformations due to temperature

* Corresponding author at: Department of Mechanical Engineering, Kinki University, 3-4-1 Kowakae, Higashi-Osaka, Osaka 577-8502, Japan
E-mail address: ochiai@mech.kindai.ac.jp (Y. Ochiai).

0955-7997/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.enganabound.2012.09.002

gradients, the inertial terms can be omitted because the characteristic frequency of elastic processes is several orders higher than that of thermal processes. Thus, in quasi-static uncoupled thermoelasticity, the governing equations for the temperature $T(q,t)$and displacements $u_i(q,t)$fields are given as [23,26]

$$\nabla^2 T - \frac{1}{\kappa}\frac{\partial T}{\partial t} = -\frac{w}{\lambda} \tag{1}$$

$$G\nabla^2 u_i + \frac{G}{1-2v}u_{j,ji} - \gamma T_{,i} = 0, \quad \gamma = 2G\frac{1+v}{1-2v}\alpha, \tag{2}$$

where $w(q,t)$ is the volumetric density of heat sources per unit time, and λ, κ, G, v, α stand for coefficient of heat conduction, thermal diffusivity, shear modulus, Poisson ratio, coefficient of linear thermal expansion, respectively. The set of governing equations should be supplemented by initial and boundary conditions.

Denoting the Laplace transforms of the field variables with respect to time by over bars, the governing equations become

$$\left(\nabla^2 - \frac{s}{\kappa}\right)\overline{T}(q,s) = -\frac{\overline{w}(q,s)}{\lambda} - \frac{T(q,0)}{\kappa} \tag{3}$$

$$G\nabla^2 \overline{u}_i(q,s) + \frac{G}{1-2v}\overline{u}_{j,ji}(q,s) - \gamma \overline{T}_{,i}(q,s) = 0 \tag{4}$$

with s being the Laplace transform parameter and $T(q,0)$ is the distribution of the initial value of temperature specified by initial conditions. Recall that we shall not develop the integral formulation in the Laplace transform domain, but it will be useful to utilize these governing equations in the development of the formulation without domain integral of temperature gradients.

3. Integral representations for solutions

Since the temperature field in the uncoupled thermoelasticity is not influenced by mechanical fields, one can solve firstly the initial-boundary value problem for the thermal fields and subsequently for mechanical fields. Starting from the governing Eqs. (1) and (A.5), one can derive in a standard way the integral representation of the temperature field:

$$c(p)T(p,t) = \kappa\int_0^t\int_\Gamma\left[\frac{\partial T(Q,\tau)}{\partial n(Q)}T^{*[1]}(p,Q,t-\tau) - T(Q,\tau)\frac{\partial T^{*[1]}(p,Q,t-\tau)}{\partial n(Q)}\right]d\Gamma\, d\tau$$
$$+\frac{\kappa}{\lambda}\int_0^t\int_\Omega w(q,\tau)T^{*[1]}(p,q,t-\tau)d\Omega\, d\tau + \int_\Omega T(q,0)T^{*[1]}(p,q,t)d\Omega \tag{5}$$

where $c(p)=1$ for interior point $p\in\Omega$, while for $p=P\in\Gamma$ it depends on the shape of the boundary geometry ($c(P)=0.5$ if Γ is smooth at P).

Similarly, from (2) and (A.14) one can get the integral representation of displacements [24,26]

$$c_{ki}(p)u_i(p,t) = \int_\Gamma [p_i(Q,t)U_{ik}(p,Q) - u_i(Q,t)T_{ik}(p,Q)]d\Gamma + \gamma\int_\Omega T(q,t)U_{ik,i}(p,q)d\Omega \tag{6}$$

where the traction vector is given by $p_i(Q,t)=n_j(Q)[c_{ijkl}u_{k,l}(Q,t) - \gamma\delta_{ij}T(Q,t)]$ with$c_{ijkl} = G(2v/(1-2v))\delta_{ij}\delta_{kl} + G(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$ for homogeneous and isotropic linear elastic medium. Note that there is no integration with respect to time in (6) because of quasi-static approximation.

From the point of view of boundary elements, certain handicap of both the derived integral formulae (5) and (6) is the appearance of domain integrals. In what follows, we shall deal with conversion of these domain integrals into boundary ones by using higher order fundamental solutions and triple reciprocity approximations of the heat sources and initial temperature.

3.1. Treatment of domain integrals. Triple-reciprocity approximations

It is well known that if the spatial distribution of the heat sources $w(q,\tau)$ were given by a harmonic function, the following conversion of the domain integral into boundary ones would be applicable:

$$\int_\Omega w(q,\tau)T^{*[1]}(p,Q,t-\tau)d\Omega = \int_\Gamma\left[w(Q,\tau)\frac{\partial T^{*[2]}(p,Q,t-\tau)}{\partial n(Q)} - \frac{\partial w(Q,\tau)}{\partial n(Q)}T^{*[2]}(p,Q,t-\tau)\right]d\Gamma,$$

where we have utilized the property given by Eq. (A.2) and the assumption $\nabla^2 w(q,\tau)=0$. Similar conversion would be applicable also to domain integral of initial temperature, if the distribution of this temperature was given by a harmonic function. In what follows, we shall assume the spatial distributions of the heat sources as well as the initial temperature without any restrictions.

Let us replace $w(q,\tau)$ by $W_1^S(q,\tau)$ whose spatial distribution is governed by

$$\nabla^2 W_1^S(q,\tau) = -W_2^S(q,\tau), \tag{7}$$

with prescribed values of heat sources $w(q,\tau)\approx W_1^S(q,\tau)$ at a set of interior and boundary points. Furthermore, the spatial distribution of $W_2^S(q,\tau)$ is assumed to be governed by the equation

$$\nabla^2 W_2^S(q,\tau) = -\sum_{m=1}^M W_3^{PA}(q_m,\tau)\delta(q-q_m), \tag{8}$$

where $W_3^{PA}(q_m,\tau)$ are unknown nodal values at interior points $q_m\in\Omega$ and $W_2^S(Q,\tau)=0$ is assumed on the boundary of the analyzed domain.

Just to remember an analogy [15,16,18–22], substitution of (7) into (8) yields

$$\nabla^4 W_1^S(q,\tau) = \sum_{m=1}^M W_3^{PA}(q_m,\tau)\delta(q-q_m) \tag{9}$$

This equation is formally the same as that for the deformation of a fictitious thin plate with point loads, but now the analyzed domain is 3D in contrast to the mid-surface of the plate. The "deformation" $W_1^S(q,\tau)$ is given, but the "forces of the point loads" $W_3^{PA}(q_m,\tau)$ are unknown and can be calculated inversely from the "deformation" $W_1^S(q,\tau)$ of the fictitious thin plate. The "bending moment" $W_2^S(Q,\tau)$ is vanishing.

Similarly, the distribution of the initial temperature $T(q,0)$ can be approximately replaced by $T_1^{0S}(q)$ which is interpolated as follows:

$$\nabla^2 T_1^{0S}(q) = -T_2^{0S}(q), \tag{10}$$

$$\nabla^2 T_2^{0S}(q) = -\sum_{m=1}^M T_3^{0PA}(q_m)\delta(q-q_m) \tag{11}$$

with $T_1^{0S}(q)$ being known at boundary nodes and at the set of interior points $q_m\in\Omega$ $(m=1,2,...,M)$, while $T_2^{0S}(Q)=0$ is assumed on the boundary of the analyzed domain.

Now, we reformulate the governing Eqs. (7) and (8) into integral equations. Making use of the polyharmonic fundamental solutions of the Laplace operator with properties (A.2), one can derive the integral equations:

$$c(P)W_1^S(P,\tau) = \sum_{g=1}^2(-1)^g\int_\Gamma\left\{W_g^S(Q,\tau)\frac{\partial T^{[g]}(P,Q)}{\partial n(Q)} - \frac{\partial W_g^S(Q,\tau)}{\partial n(Q)}T^{[g]}(P,Q)\right\}d\Gamma$$
$$-\sum_{m=1}^M T^{[2]}(P,q_m)W_3^{PA}(q_m,\tau) \tag{12}$$

$$c(P)W_2^S(P,\tau) = \int_\Gamma\left\{\frac{\partial W_2^S(Q,\tau)}{\partial n(Q)}T^{[1]}(P,Q) - W_2^S(Q,\tau)\frac{\partial T^{[1]}(P,Q)}{\partial n(Q)}\right\}d\Gamma$$
$$+\sum_{m=1}^M T^{[1]}(P,q_m)W_3^{PA}(q_m,\tau) \tag{13}$$